Land Subsidence and Cracking
Due to Ground-Water Depletion
by Herman Bouwer

ABSTRACT
Subsidence of the land surface due to ground-water overdraft is caused by an increase in the intergranular pressure in unconsolidated aquifers and other underground materials. For unconfined aquifers, this increase is the result of a loss of buoyancy of solid particles in the zone dewatered by the falling water table. For confined aquifers, increases in intergranular pressure are caused by decreases in the upward hydraulic pressure against the bottom of the upper confining layer, due to a drop in piezometric surface. Compression of layers in which the intergranular pressure is increased can be calculated with elastic or logarithmic theory. Sample calculations yield rates of subsidence that agree with those observed, i.e., about 5 to 50 cm (2 to 20 inches) per 10 m (33 ft) drop in ground-water level. Ground-water depletion can also produce surface cracks, particularly above discontinuities in bedrock depth along the periphery or in other parts of subsiding basins. Calculations based on the rotating-slab theory show that the initial surface width of such cracks is about 1 cm (0.5 inch), which agrees with field observations.

INTRODUCTION
Downward movement or subsidence of the land surface is an important environmental consequence of ground-water overdraft. It is caused by compression of underground materials due to declining water tables or piezometric surfaces. In addition, initiation or acceleration of lateral flow of ground water can cause lateral compression of the aquifer and, hence, lateral movement of the land surface, due to an increase in the seepage force or frictional drag exerted by the flowing water on the solid particles. Theoretically, any flow or overdraft; of ground water in unconsolidated material should produce some movement of the land surface. This movement normally is quite small, but it can become significant where underground materials are thick and/or compressible and ground-water levels decline appreciably. Recorded subsidences range from a few centimeters (about 1 inch) to almost 10 m (33 ft), as shown in Table 1. Subsidence rates range from about 1 to 50 cm per 10-m drop in ground-water level (0.01 to 0.5 ft per 10-ft drop), depending on thickness and compressibility of the formations. Lateral movement of the land surface of several meters has been reported in conjunction with removal of oil and gas. Nonuniform subsidence, which may result from different rates of ground-water declines or differences in compressibility of underground formations, can also produce cracks or fissures in the earth's surface.

Land subsidence has increased flood hazards (Venice, Baytown-Houston) and has caused cracking of buildings, misalignment of bridge abutments, damage to roads, railways, storm sewers or other underground pipelines, collapse of well casings, and reversal of gradients of irrigation canals or other conduits. Land subsidence due to ground-water overdraft is essentially irreversible. It can be stopped by halting declines in ground-water levels (combined with ground-water replenishment if necessary to prevent residual compression of clay layers). However, rebound of the land surface normally is insignificant, even if ground-water levels are restored to presubsidence heights.

INTERGRANULAR PRESSURE
The basic cause of subsidence and lateral movement of the land surface is an increase in the intergranular pressure of the underground materials.
Table 1. Examples of Land Subsidence (1 m = 3.28 feet)

<table>
<thead>
<tr>
<th>Location</th>
<th>Maximum Subsidence, m</th>
<th>Period</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venice, Italy</td>
<td>0.15</td>
<td>1930-1973</td>
<td>Gambolati and Freeze, 1973</td>
</tr>
<tr>
<td>Mexico City</td>
<td>8</td>
<td>1938-1968</td>
<td>Poland, 1969</td>
</tr>
<tr>
<td>Tokyo, Osaka</td>
<td>4</td>
<td>1928-1943</td>
<td>Poland, 1969</td>
</tr>
<tr>
<td>Taipei</td>
<td>1</td>
<td>1960</td>
<td>Poland, 1969</td>
</tr>
<tr>
<td>London</td>
<td>0.18</td>
<td>1865-1931</td>
<td>Poland and Davis, 1969</td>
</tr>
<tr>
<td>Baton Rouge, La.</td>
<td>0.3</td>
<td>Since 1890</td>
<td>Davis and Rollo, 1969</td>
</tr>
<tr>
<td>Houston-Baytown</td>
<td>2.7</td>
<td></td>
<td>Jones and Warren, 1976</td>
</tr>
<tr>
<td><strong>Irrigation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>San Joaquin Valley, Calif.</td>
<td>8.5</td>
<td></td>
<td>Lofgren, 1969</td>
</tr>
<tr>
<td>Santa Clara Valley, Calif.</td>
<td>4</td>
<td></td>
<td>Poland, 1969</td>
</tr>
<tr>
<td>Eloy Area, Arizona</td>
<td>2.3</td>
<td>1948-1967</td>
<td>Schumann and Poland, 1969</td>
</tr>
<tr>
<td><strong>Geothermal</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wairakei, New Zealand</td>
<td>4</td>
<td></td>
<td>Axtmann, 1975</td>
</tr>
<tr>
<td><strong>Oil</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wilmington, Calif.</td>
<td>9*</td>
<td></td>
<td>Grant, 1954</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mayuga and Allen, 1969</td>
</tr>
</tbody>
</table>

*In addition, lateral displacements up to 3.7 m were observed.

Such an increase causes a vertical and/or lateral compression of underground formations. The intergranular or effective pressure is the pressure that is transmitted by the individual grains of the underground material at their contact points. For tectonically relaxed systems, the vertical intergranular pressure is defined as

\[ P_i = P_t - P_h \]  

where:
- \( P_i \) = intergranular pressure,
- \( P_t \) = total pressure, and
- \( P_h \) = hydraulic pressure.

This relation can readily be visualized by assuming an imaginary horizontal plane at some depth in the aquifer. The vertical load on this plane is the weight of everything that is above it. However, there is also an upward force acting on the plane, due to the hydraulic pressure of the ground water. The difference between the two loads is the net load, which must be carried by the individual grains of the material.

The total pressure at a given depth is calculated as the weight per unit of horizontal area of every-thing (solids as well as liquids) that lies above that depth. For the unconfined aquifer in Figure 1 with physical properties as shown in Table 2, for example it can be calculated that \( P_t \) at the initial water table is 3.84 kg/cm² (54.6 pounds per square inch, or psi), while at the bottom of the aquifer \( P_t \) is equal to 3.84 + 16.96 = 20.8 kg/cm² (296 psi). This yields the \( P_{t1} \)-line in Figure 1. The hydraulic pressure increases linearly from zero at the water table to 8 kg/cm² (114 psi) at the bottom of the aquifer, yielding the \( P_{h1} \)-line in Figure 1. Above the water table, \( P_h \) is negative. Actual values of \( P_h \) in this region, however, are difficult to predict because they depend on vertical flow rates and water contents in the vadose zone. For this reason, \( P_h \) above the water table normally is considered zero.

The horizontal distance between the \( P_{t1} \)-line and \( P_{h1} \)-line in Figure 1 is equal to \( P_{h1} \), in accordance with Equation (1). Assuming that the water table has dropped from a depth of 20 m to
50 m (65.6 to 164 ft) and that the water content of the dewatered zone is 10%, like in the original vadose zone, \( P_1 \) and \( P_2 \) after the water-table drop can be calculated similarly, yielding the \( P_{12} \) and \( P_{22} \)-curves indicated with dashed lines in Figure 1. The horizontal distance between the dashed lines is greater than that between the solid lines, indicating that the water-table drop has caused an increase in \( P_1 \). At 80-m (262.4-ft) depth, for example, \( P_1 \) has increased from 10.6 to 13 kg/cm\(^2\) (151 to 185 psi). The increase in \( P_1 \) is uniform at 2.4 kg/cm\(^2\) (34.1 psi) for the entire aquifer below the new water table, but it increases linearly in the dewatered zone from zero at the old water table to 2.4 kg/cm\(^2\) at the new water table (dotted line in Figure 1).

The increase in \( P_1 \) is due to the loss of buoyancy of the solids in the dewatered zone (from 20- to 50-m depth). Since the porosity of the material was taken as 30% (Table 2), 1 cm\(^3\) of the material contains 0.7 cm\(^3\) solids, which upon loss of buoyancy become 0.7 g heavier. Adding the weight of the water remaining behind in the dewatered zone, which at the 10% water content amounts to 0.1 g/cm\(^3\), to the weight increase due to loss of buoyancy yields a total effective weight increase of 0.8 g/cm\(^3\) of the material in the dewatered zone. Since the dewatered zone is 30 m thick, this amounts to a total weight increase of 2.4 kg/cm\(^2\), which is the same as the increase in \( P_1 \) calculated from the \( P_{12} \) and \( P_{22} \)-lines before and after the water table drop. Thus, for unconfined aquifers, subsidence is caused by compaction of underground materials due to an increase in intergranular pressures resulting from the loss of buoyancy of solids in the zone dewatered by the declining water table.

For confined aquifers, increases in intergranular pressures are caused by decreases in the upward hydraulic pressure against the bottom of the upper confining layer, due to a declining piezometric surface. The decrease in upward hydraulic pressure then effectively causes an increase in the overburden weight. This is illustrated in Figure 2, which shows that the increase in \( P_1 \) is constant with depth in the aquifer and equal to the reduction in hydraulic pressure due to the drop in piezometric surface. If the upper confining layer contains water in a continuous matrix and there is no ground water above the confining layer, the increase in \( P_1 \) in that layer will decrease linearly from the aquifer value at the bottom to zero at the top (Figure 2). The values of \( P_1 \) in Figure 2 were calculated on the assumption that the confined aquifer and the upper confining layer had the same physical properties as the unconfined aquifer in Table 2, and that the vadose zone had the same properties as shown in Table 2. Since there is no dewatering of pore space by a drop in piezometric surface, the \( P_1 \)-line is not affected.

**CALCULATION OF SUBSIDENCE**

The calculation of compression of layers in which \( P_1 \) is increased is based on how the porosity, or rather the void ratio as commonly used in soil mechanics, of the layer is reduced by an increase in \( P_1 \). The relation between void ratio and \( P_1 \) for a certain unconsolidated material can be determined in the laboratory, by applying increasing vertical loads to a sample and determining the resulting decrease in void ratio from measurements of the compaction of the sample with an extensometer. Curves of the void ratio \( e \) versus \( P_1 \) (Figure 3) generally show that the rate of decrease in \( e \) diminishes with increasing \( P_1 \). Also, removal of the load causes some rebound of the material, but not to the original position (point A in Figure 3). Resumption of the load produces hysteresis in the curve and when the curve joins the original curve where the load was interrupted, a discontinuity may be observed (point B in Figure 3).

The relation between the compression \( S_u \) of a certain layer of thickness \( Z_1 \) due to a reduction in void ratio from \( e_1 \) to \( e_2 \) in conjunction with an effective pressure increase from \( P_{11} \) to \( P_{12} \) can be derived as (see, for example, Bouwer, 1978)

\[
S_u = Z_1 \frac{e_1 - e_2}{e_1 + 1} \tag{2}
\]

To calculate subsidence of the land surface, Equation (2) is applied to the various formations...
Solving this equation for $S_u$ yields

$$S_u = \frac{P_2 - P_1}{P_2 - P_1} Z_1$$

(3)

The modulus of elasticity can be evaluated from curves of $S_u$ versus $P_1$. Combining Equations (2) and (3) shows that

$$E = \frac{S_u}{c_1 + 1}$$

(4)

where $S_u$ is the slope of the curve of $S_u$ versus $P_1$, and $c_1$ is the coefficient of $S_u$. The value of $c_1$ is constant and equal to the modulus of elasticity $E$. The modulus of elasticity can be evaluated from curves of $S_u$ versus $P_1$. Combining Equations (2) and (3) shows that

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(5)

This equation is valid only for $P_1$ values below the range where $e$ varies linearly with $P_1$. Values of $P_1$ above this range cannot be used, since the relationship between $e$ and $P_1$ becomes non-linear.

(6)

The modulus of elasticity $E$ is expressed in terms of the stress-strain curve of the material, and is calculated as

$$E = \frac{S_u}{c_1 + 1}$$

(7)

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(8)

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Table 3. Orders of Magnitude of $E$ (1 kg/cm$^2$ = 14.2 psi) and $C_u$ (dimensionless) for Unconsolidated Materials (as compiled from various sources by Bourner, 1978)

<table>
<thead>
<tr>
<th></th>
<th>$E$ in kg/cm$^2$</th>
<th>$C_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense gravel and sand</td>
<td>2,000-10,000</td>
<td>0.005</td>
</tr>
<tr>
<td>Dense sand</td>
<td>500-2,000</td>
<td>0.01</td>
</tr>
<tr>
<td>Loose sand</td>
<td>100-200</td>
<td>0.05</td>
</tr>
<tr>
<td>Dense clay and silt</td>
<td>100-1,000</td>
<td>0.05</td>
</tr>
<tr>
<td>Medium clay and silt</td>
<td>50-100</td>
<td>0.1</td>
</tr>
<tr>
<td>Loose clay</td>
<td>10-50</td>
<td>0.3</td>
</tr>
<tr>
<td>Peat</td>
<td>1-5</td>
<td>0.2-0.8</td>
</tr>
</tbody>
</table>

Coefficient $C_u$ (see, for example, Colijn and Potma, 1944), reducing Equation (8) to

$$S_u = Z_1 C_u \log \frac{P_{12}}{P_{11}} \quad (9)$$

To apply Equation (4) or Equation (9), one must know $E$ or $C_u$. These coefficients can be evaluated from laboratory tests as discussed earlier, or they can be determined from measured subsidence or compression in response to a certain drop of water table or piezometric surface.

Approximate ranges for $E$ and $C_u$ are listed in Table 3. These values should be considered only as general ones and should not be used to predict subsidence for actual situations. Such predictions should be based on local values of $E$ or $C_u$. The data in Table 3 show that the compressibility of granular materials increases with decreasing particle size. Organic deposits are the most compressible. By way of comparison, $E$ of water is about 20,000 kg/cm$^2$ (284,460 psi).

Examples of Using Equations (4) and (9)

Assuming that the unconfined aquifer in Figure 1 consists of dense sand with $E = 1,000$ kg/cm$^2$ for the dewatered zone from 20 to 30 m and $E = 3,000$ kg/cm$^2$ for the lower 50 m of the aquifer (the higher value of $E$ accounts for the fact that $E$ increases with increasing $P_{11}$) and taking the average $P_{11}$ and $P_{12}$-values for each zone, Equation (4) yields $S_u = 0.036$ m for the 20- to 50-m zone and $S_u = 0.024$ m for the 50- to 100-m zone, or a total subsidence of 0.06 m. Taking $C_u = 0.01$ for both zones (the logarithmic theory automatically takes care of the decreasing compressibility of the material with increasing depth), Equation (9) yields $S_u = 0.026$ m for the 20- to 50-m zone and 0.028 m for the 50- to 100-m zone, or a total subsidence of 0.054 m. Thus, the subsidence is about 2 cm per 10-m water-table drop. This is at the low end of the range observed in practice, as can be expected for an aquifer consisting of dense sand with no clay layers. If the aquifer below the 50-m depth in the example had contained beds of compressible clay with a total thickness of 20 m and an $E$-value of 30 kg/cm$^2$, the compression due to a $P_{11}$-increase of 2.4 kg/cm$^2$ would have been 1.6 m. The resulting subsidence from the clay layers alone would have produced a subsidence of 0.53 m per 10-m water-table drop, which is at the top of the range of values observed in practice.

For the confined aquifer in Figure 2, taking $E$ as 100 kg/cm$^2$ for the upper confining layer (clay) and as 1,000 kg/cm$^2$ for the aquifer (sand) and again using the average $P_{11}$-values for each layer, Equation (4) yields $S_u = 0.1$ m for the confining layer and $S_u = 0.04$ m for the aquifer. Taking $C_u$ as 0.1 for the confining layer and 0.01 for the aquifer, Equation (9) yields $S_u$-values of 0.082 and 0.029 m, respectively. Thus the total subsidence is 0.14 and 0.112 m, respectively, for these solutions, which yields a subsidence of about 0.06 m per 10-m drop in piezometric surface. If compression of the aquifer and of the upper confining layer is the only way in which water is yielded, the storage coefficient of the aquifer would be 0.006. Since $E$ for water (i.e., 20,000 kg/cm$^2$) is several orders of magnitude higher than $E$ for unconsolidated materials, expansion of water contributes insignificantly to storage coefficients of confined aquifers consisting of such materials. Only when the water is saturated with certain dissolved gases which go out of solution upon pressure reduction, will decompression of water contribute significantly to storage coefficients.

The treatment of subsidence in this paper is a simple and elementary way of predicting final subsidence due to ground-water overdraft. Actually, the subsidence rate may lag behind the ground-water decline rate, because it may take time for water to be squeezed out of clay layers being compressed. If the clay is tight and the layers are thick, decades may pass before the excess pore water has been released and the increase in effective overburden pressure is entirely carried by the clay particles at their contact points. In such cases, subsidence can continue many years after ground-water decline has stopped. Such residual subsidence can be avoided by restoring ground-water levels to original elevations. The rate of compression of a clay layer in response to a load increase varies linearly with the hydraulic conductivity of the layer and inversely with the square of the thickness of the layer (Terzaghi and Peck, 1948).
crack formation

Under certain conditions, ground-water over- 
draft and resulting subsidence can cause fissures in 
the land surface. Most of these cracks appear to 
be simple tensile breaks with no vertical or 
lateral offset. Schumann and Poland (1969) 
reported for a ground-water basin in south-central 
Arizona that such cracks initially were narrow, 
probably less than 2 cm (0.8 in.) wide and about 
1 or 2 km (1 mile) long. The cracks appeared to be 
concentrated along the periphery of the subsiding 
basin and ran approximately parallel to the 
surrounding mountains. Most cracks ran normal to 
the surface-drainage pattern, causing them to 
intercept runoff, erode, cave, and become 
interconnected. Some of the resulting “gullies” 
eventually were several meters (about 10 ft) wide, 
5 to 10 m (16 to 33 ft) deep, and more than 10 km 
(6 miles) long (Figure 4). The cracks initially may 
have been deeper than their final 5- to 10-m (16 to 
33 ft) depth, because caving and accumulation of 
soil material may have filled or covered the deeper 
portions. Vertical extension of cracks below the 
secondary fill has been observed by excavation and 
drilling in the fissures (personal communication, 
R. H. Raymond, U.S. Bureau of Reclamation, 
Phoenix, Arizona). Schumann and Poland also 
reported that the trends of several of the fissures 
conformed with linear zones of steep gravity 
gradients adjacent to mountain ranges. These steep 
gradients may indicate buried fault scarps.

Formation of tensile cracks above buried 
fault scarps at the periphery of a subsiding basin 
could be the result of a linearly increasing 
subsidence from the edge of the basin to the center. 
Such increasing subsidence then could produce a 
rotational movement of the upper slab of alluvial 
fill around the top of the underground scarp 
(Figure 5). An approximately linear increase in 
subsidence from the edge of the basin to the center 
can be caused by greater withdrawal of ground 
water in the center portion of the basin. Also, deep 
clay beds and other compressible formations may 
be thickest in the center of the basin and become 
thinner towards the edges. Under these conditions, 
compression of the deeper clay beds and other 
materials below the water table tends to increase 
from the edge of the basin to the center, causing 
the overlying alluvium to rotate from CE to CF 
above the compressing layers and from AB to AD 
at the surface, and a crack to be formed above the 
scarp (Figure 5). The pivot point for the rotation 
is where the compressible deposits in Figure 5 run 
out against the underground scarp. If there is no 
compression at that point, the fissure will be a 
simple, tensile break. The same will be true if 
compressible layers on both sides of the scarp 
undergo equal compression. If, however, 
compression takes place on the basin side of the 
underground scarp but not on the mountain side, 
the fissure may also show a vertical offset. Assuming 
that AB in Figure 5 is about 10 km (6 miles), BD 
is 2 m (6.6 ft) and AC is 50 m (164 ft), application 
of similar triangles to the crack and triangle ABD 
shows that initially the crack is 1 cm (0.4 inch) 
wide at the surface, which is on the same order as 
observed in the field (Schumann and Poland, 1969).

The rotating-slab theory may also explain

![Fig. 4. Eroded earth crack in caliche-cemented alluvium west of Chandler, Arizona.](image)

![Fig. 5. Schematic of crack development above buried scarp due to rotation of slab ABEC.](image)
formation of cracks above underground ridges in basement rock (Figure 6). Since the areas above such ridges tend to be less suitable for well sites because of the limited depth, ground-water withdrawal tends to be concentrated at some distance on both sides of the ridge where the alluvial materials are thicker. This could produce a linearly increasing subsidence away from the ridge, causing the overlying slabs on each side of the ridge to rotate similar to the rotation in Figure 5, and cracks to form above the ridge.

It may also seem plausible that earth cracks could be formed by a stretching of the land surface. If the subsidence is purely vertical, AD in Figure 5 is longer than AB. However, this normally gives such a narrow crack (0.2 mm (0.008 inch) if AB = 10 km (6 miles) and BD = 2 m (6.6 ft)), that it is not a likely explanation. Another possible reason for crack formation is differential lateral movement of the land surface. If ground-water withdrawal is concentrated in a certain part of a ground-water basin, lateral flow rates of ground water will be highest in and near that area. Because of horizontal seepage forces, this will then produce more lateral compression, and, hence, more lateral movement of the land surface near the area of pumping than at some distance away. The differential lateral movement then could produce cracks that may run more or less concentric in or around the area of pumping. Cracks could also be concentrated where there are discontinuities in underground materials, such as alluvial fans with coarse-textured materials and little tensile strength grading into finer, more plastic valley fills. Another possible mechanism for crack formation, proposed by Holzer (1976), is differential lateral movement by horizontal shrinkage of the sediments in the zone dewatered by declining ground-water levels.

Earth cracking is a surface manifestation of ground-water overdraft that can be caused by a variety of subsurface conditions that sometimes may be quite complex. While the models and mechanisms presented in the previous paragraphs may give some of the reasons for earth cracking, true causes can only be evaluated by intensive field and subsurface investigations. Considering the increased use of ground water and the potential damage of land subsidence and particularly earth cracking due to ground-water overdraft, additional research to determine causes and to predict where earth cracks may develop is very much needed.

REFERENCES


