

A Better Numeration System

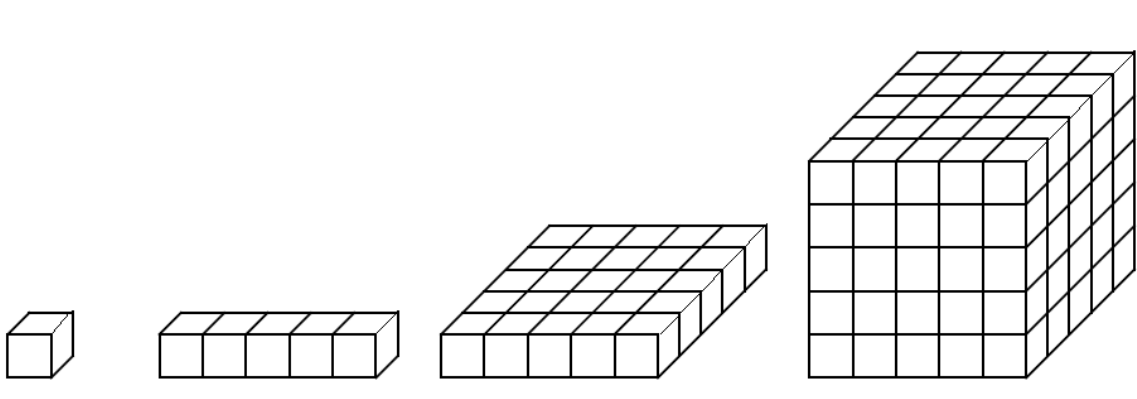
After some time, there was a conference called so that the mathematicians could get together to share their ideas.

Excitement was high as the Xmanian mathematicians began to arrive at the conference. Several proposals for new number systems were on the program, but the one that generated the most excitement was a system which only used the symbols "O", "A", "B", "C", and "D". It was rumored that this system solved the "many" problem: you could count as high as you wanted and you could identify exactly how many items there were in a set. To everyone's amazement, it was further rumored that the mathematician had developed algorithms for adding, subtracting, multiplying and dividing within the new system.

As the day of the presentation dawned, mathematicians from all over Xmania began to fill the auditorium. Soon all the seats were filled, and in no time, the aisles were also packed. A respectful hush fell across the crowd as the presider stepped up to the podium to introduce the inventor of the new system.

"It is my sad duty to announce that our most esteemed colleague and friend -- the inventor of the system about which we are all so excited -- passed away last night. She was very old, and she quietly slipped away from us in her sleep. She was a kind person, a first rate mathematician, a respected colleague, and a great teacher. We will miss her deeply."

"We have not been able to locate any written record of the system she was about to share with us. The only thing we have to provide us with a clue about her work is a set of blocks. It is my sincere hope that we, the mathematicians of Xmania, will be able to work together to reconstruct the system that our dear colleague had planned to share with us."



Your research group has decided to take on the challenge to reconstruct the numeration system that went the way of the famous mathematician. Construct a numeration system which meets *all* the characteristics of the mathematician's system. This means that the system:

- (1) bears a clear and distinct relationship to the set of blocks she left behind;
- (2) uses only the symbols 0, A, B, C, and D;
- (3) allows us to count as high as we want; and
- (4) allows us to *uniquely* identify the number of items in any collection.

IMPORTANT NOTE: To keep from totally confusing the Xmanians, you should keep the meaning of the symbols 0, A, B, C, D the same as they've always been. This means that:

0	represents	No items
A	represents	●
B	represents	● ●
C	represents	● ● ●
D	represents	● ● ● ●

You're going to be telling them about a new numeration system, but you can refer to the old system to help them through the transition. *Initially* referring to what they already know is a good teaching strategy; make a connection to what they know as you're introducing the new system.

Include any information (i.e.;illustrations, examples, explanations, etc.) that would be necessary for a novice to:

1. learn how to count in your system;
2. learn how to symbolically represent quantities in your system;
3. *understand* why your strategy makes sense;
4. be convinced that your system will allow her to use only the symbols 0, A, B, C, and D to *uniquely* identify the number of items in any collection; and
5. be convinced that your system *will work reliably* to count as high as she wants.

Remember: Although you may know how to count in the way that Earthlings count, Xmanians know nothing about the Earthling counting system. Be sure that you use mathematical ideas that Xmanians can understand (which is not Earthling!) when you explain the new Xmanian counting system.