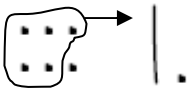
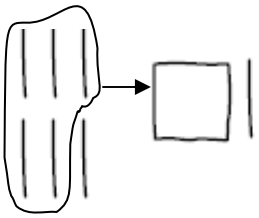
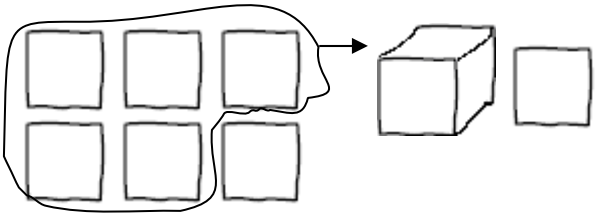
1. a. 

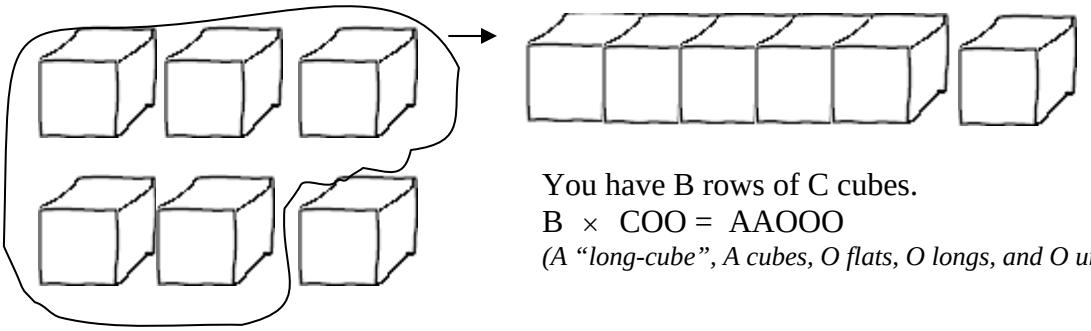
You have B rows of C units
B × C = AA
(A long and A unit)

b. 

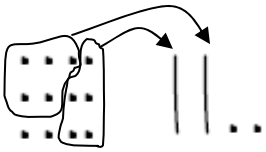
You have B rows of C longs.
B × CO = AAO
(A flat, A long, and O units)

c. 

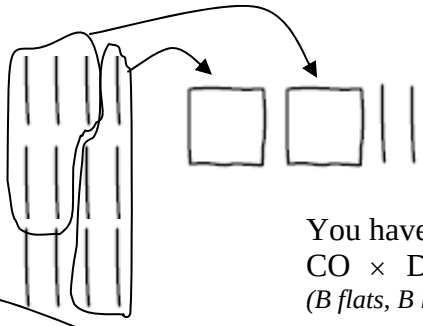
You have B rows of C flats.
B × COO = AAOO
(A cube, A flat, O longs, and O units)



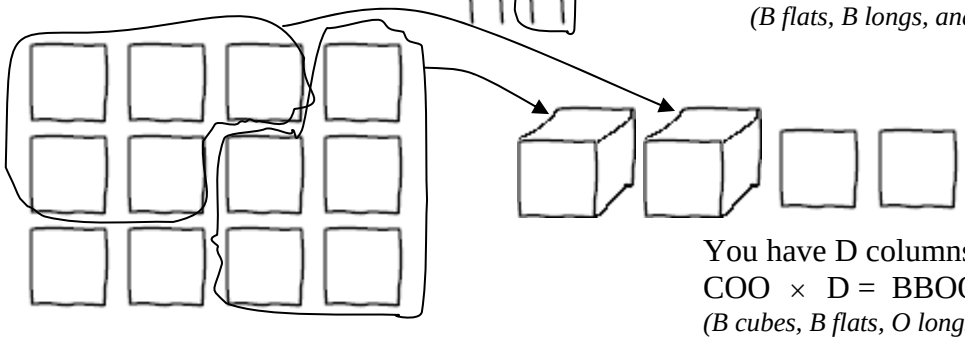
You have B rows of C cubes.
B × COO = AAOOO
(A "long-cube", A cubes, O flats, O longs, and O units)

2. a. 

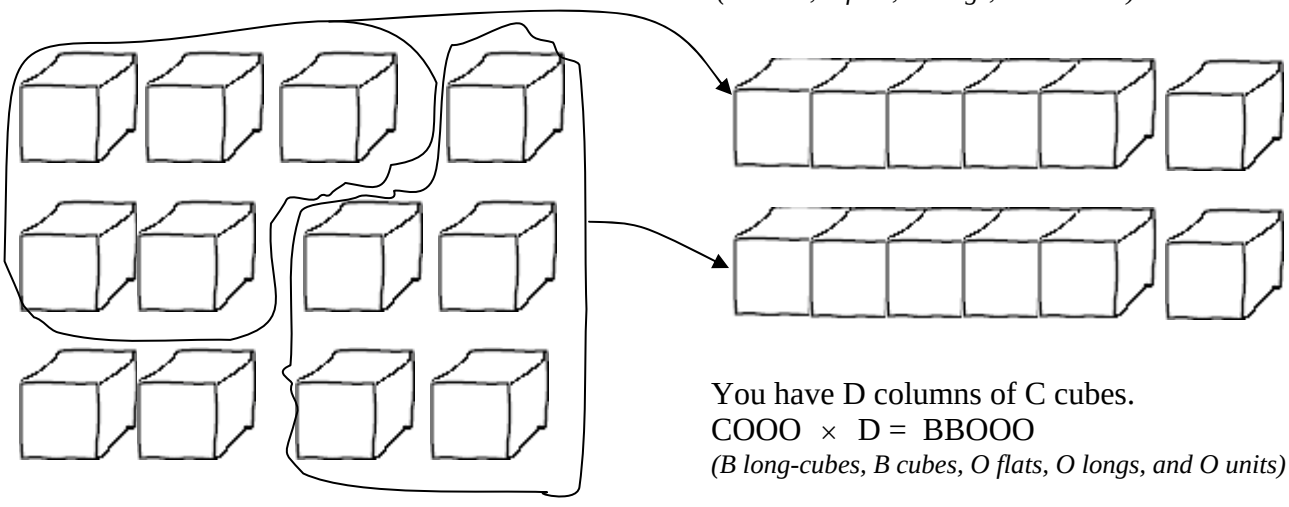
You have D columns of C units
C × D = BB
(B longs and B units)

b. 

You have D columns of C longs.
CO × D = BBO
(B flats, B longs, and O units)

c. 

You have D columns of C flats.
COO × D = BBOO
(B cubes, B flats, O longs, and O units)

d. 

You have D columns of C cubes.
COOO × D = BBOOO
(B long-cubes, B cubes, O flats, O longs, and O units)

3. a.

You have A row of C units
which is already a minimal collection.
A × C = C

b. 

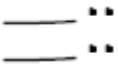
AO is A long and O units.
You have C longs which is already a minimal collection.
C × AO = CO
(C longs and O units)

c. 

AOO is A flat, O longs and O units.
You have C flats which is already a minimal collection.
C × AOO = COO
(C flats, O longs and O units)

d. 

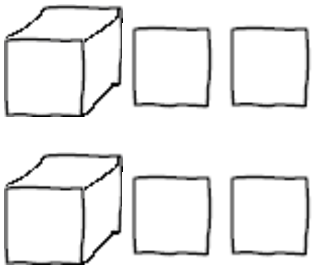
AOOO is A cube, O flats, O longs and O units.
You have C cubes which is already a minimal collection.
C × AOOO = COOO
(C cubes, O flats, O longs and O units)

4. a. 

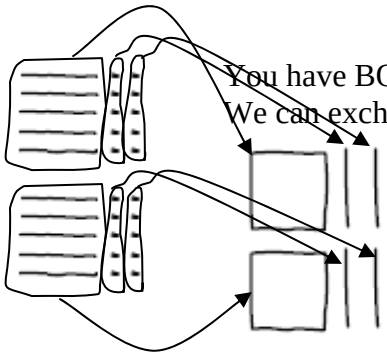
You have B rows of AB (which is A long and B units).
This makes B longs and D units which is already a minimal collection.
AB × B = BD
(B longs and D units)

b. 

You have B rows of ABO (which is A flat, B longs, and O units).
This makes B flats and D longs which is already a minimal collection.
ABO × B = BDO
(B flats, D longs and O units)

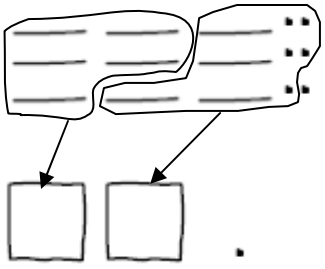
c. 

You have B rows of ABOO (which is A cube, B flats, O longs, and O units).
This makes B cubes and D flats which is already a minimal collection.
ABOO × B = BDOO
(B cubes, D flats, O longs and O units)

d. 

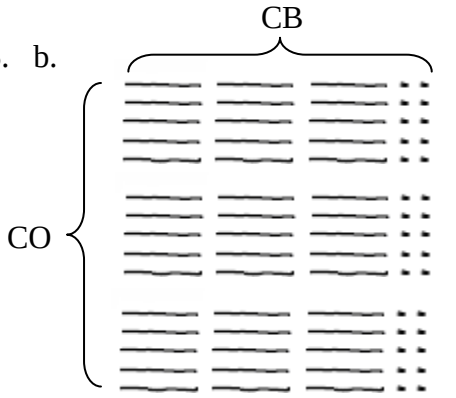
You have BO (or B hands) of AB, which is A long and B units.
We can exchange each set of AO longs for A flat, and each set of AO units for A long:

This is a minimal collection of B flats, D longs, O units or BDO.
AB × BO = BDO
Compare with 4b.

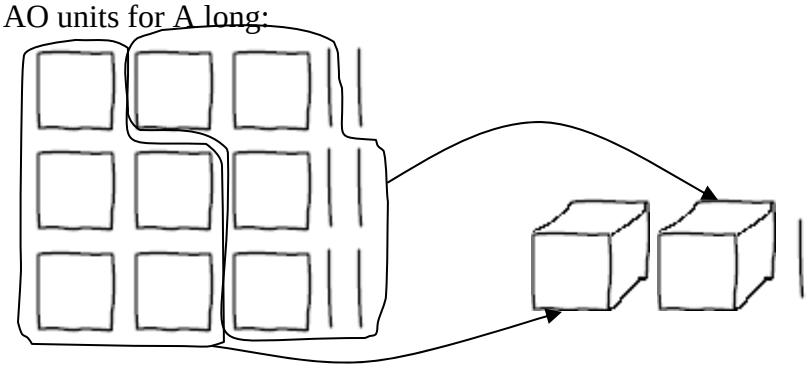
5. a. 

You have C rows of CB, which is C longs and B units.
Exchange hand longs for a flat, and D longs and hand units
for another flat, giving B flats, O longs, and A unit, so
CB × C = BOA

5. b.

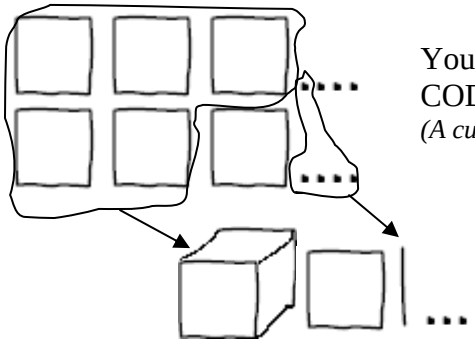


You have CO (or C hands) of CB, which is C longs and B units. We can exchange each set of AO longs for A flat, and each set of AO units for A long:



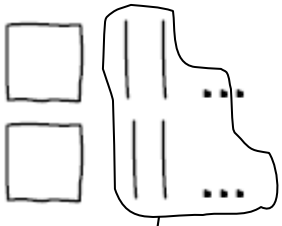
The result is B cubes, O flats, A long and O units.
CB × CO = BOAO

6.

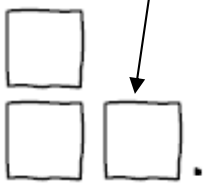


You have B rows of COD (which is C flats, O longs and D units).
COD × B = AAAC
(A cube, A flat, A long and C units)

7.



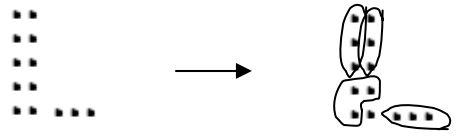
becomes:



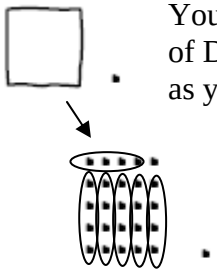
You have B rows of ABC (which is A flat, B longs, and C units). This makes C flats, O longs, and A unit after exchanges.
ABC × B = COA

8.

You have BC, which is B longs and C units, and must take away as many groups of C as possible, so regroup the B longs into BO units. Then circle as many groups of C as you can:



Therefore you have D groups of C with A remaining, or BC ÷ C = D R A

9. 

You have AOA, which is A flat, O longs and A unit, and must take away as many groups of D as possible, so regroup the A flat into AOO units. Then circle as many groups of D as you can:

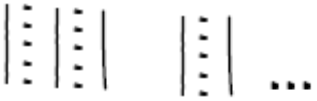
Therefore you have AA groups of D with B units remaining, so AOA ÷ D = AA R B

10. 

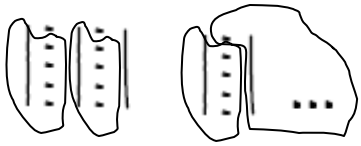
You have ACC which is A flat, C longs, and C units, and must take away as many groups of AD as you can, so regroup

becomes

which becomes



We can remove D groups which contain A long and D units, leaving AB remaining.



Therefore ACC ÷ AD = D R AB

11. To multiply by "hand", put a zero at the end.
What is B× AO? It's BO and there you go.

What is B × CO? It's B × CO or B × C × AO. Find B × C and multiply by hand.
AO × AO is also fun. It's AOO and then you're done.
AOO × AO is easy too. It's AOOO and then you're through.
What is C × BOO? It's (C × B) × AOO, so find C × B first, then multiply by AO and AO again.

The above is modeled after *The Best of Times* by Greg Tang.
See www.gregtang.com for great books.

Recognize any properties used?
What is COO × BO? Write COO as (C × AOO). Write BO as (B × AO).
Then COO × BO = (C × AOO) × (B × AO) = (C × AOO) × (B × AO)
= (AOO × C) × (B × AO)
= AOO × (C × B) × AO
= AOO × AO × (C × B)
= AO× AO × AO × (C × B)
So multiply (C × B) and then multiply by AO, then by AO, then by AO