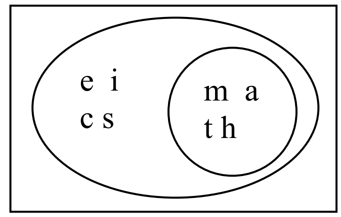
Sets as a Basis for Whole Numbers

A **set** is a collection of objects and the objects are called **elements** or **members** of the set.
The **empty set** or **null set**, written { } or $\varnothing$, is the set without any members, i.e. the set of all U.S. states bordering Antarctica is the empty set.
A nonempty set is **finite** if it can have its elements listed (where the list eventually ends), while an **infinite** set goes on without end, i.e., the set of integers $\{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$ is infinite.
The **cardinality** of a finite set *A*, written $n(A)$, is how many members it has.
What is *S* if the set *S* consists of the letters in the word *mathematics*? $S =$
How many members does it has? $n(S) =$

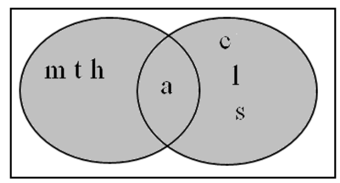
Subset of a Set: $A \subseteq B$ and Proper Subset of a Set: $A \subset B$

Set *A* is a said to be a **subset** of *B*, written $A \subseteq B$, if and only if every element of *A* is an element of *B*.
Set *A* is a said to be a **proper subset** of *B*, written $A \subset B$, if $A \subseteq B$ and there is an element of *B* that is not in *A*.



Example: The set of the original thirteen colonies is a proper subset of the set of all U.S. states.
The set of letters in the word *math* is a proper subset of the set of letters in the word *mathematics*. $\{m, a, t, h\} \subset \{m, a, t, h, e, i, c, s\}$

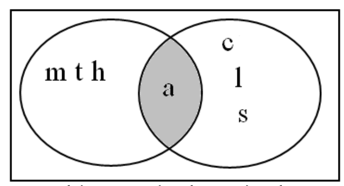
Union of Sets: $A \cup B$



The **union** of two sets *A* and *B*, written $A \cup B$, is the set of all elements belonging to either *A* or to *B* or both.
What letters are in the word *math* or in the word *class*?
The union helps us answer that question.

If $A = \{m, a, t, h\}$, $B = \{c, l, a, s\}$, then $A \cup B = \{m, a, t, h, c, l, s\}$

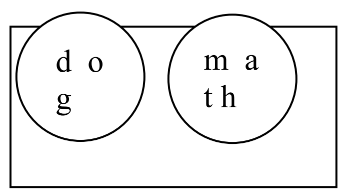
Intersection of Sets: $A \cap B$



The **intersection** of two sets *A* and *B*, written $A \cap B$, is the set of all elements common to sets *A* and *B*.
What letters are in the word *math* and in the word *class*?
The intersection helps us answer that question.

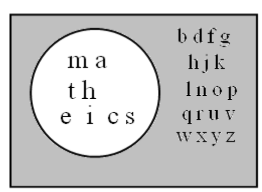
If $A = \{m, a, t, h\}$, $B = \{c, l, a, s\}$, then $A \cap B = \{a\}$

Two sets are **disjoint** if they have no elements in common.
What letters are in the word *math* and in the word *dog*?



Complement of a Set: $\overline{A}$

The **complement** of a set *A*, written $\overline{A}$, is the set of all elements in the universe that are not in *A*.
If the universe is the set of the 26 letters of our alphabet, the complement of the set of the letters in the word *mathematics* is the set of the letters of the alphabet not in the word *mathematics*.



<u>Exercises:</u>

Suppose $A = \{0, 1\}$ $B = \{1, 2, 3\}$, $C = \{0, 1, 2\}$ Sketch a Venn Diagram to illustrate their relationship.
Insert the appropriate symbol $\in$, $\notin$, $\subseteq$, $\subset$, or $\not\subseteq$ in the blank to make a true statement.

2 ____ B	A ____ B	C ____ $A \cup C$
2 ____ C	A ____ C	A ____ $A \cap C$

Closure Property for Addition:

A set of numbers is "closed under addition" if the sum of any two elements (including themselves) is also in that set.
Which of the sets *A*, *B*, and *C*, if any, are closed under addition?
Similarly, which of the sets *A*, *B*, and *C*, if any, are closed under subtraction?
Which of the sets *A*, *B*, and *C*, if any, are closed under multiplication?
Which of the sets *A*, *B*, and *C*, if any, are closed under division?