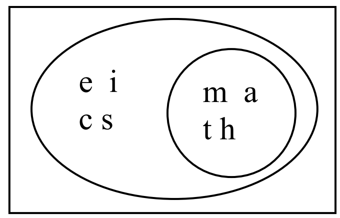
Sets as a Basis for Whole Numbers
Revised

A **set** is a collection of objects and the objects are called **elements** or **members** of the set.
The **empty set** or **null set**, written { } or $\varnothing$, is the set without any members, i.e. the set of all U.S. states bordering Antarctica is the empty set.
A nonempty set is **finite** if it can have its elements listed (where the list eventually ends), while an **infinite** set goes on without end, i.e., the set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is infinite.
The **cardinality** of a finite set *A*, written $n(A)$, is how many members it has.
What is *S* if the set *S* consists of the letters in the word *mathematics*? $S = \{m, a, t, h, e, i, c, s\}$
How many members does it has? $n(S) = 8$

Subset of a Set: $A \subseteq B$ and Proper Subset of a Set: $A \subset B$

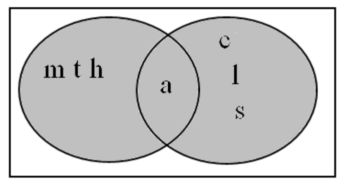
Set *A* is a said to be a **subset** of *B*, written $A \subseteq B$, if and only if every element of *A* is an element of *B*.

Set *A* is a said to be a **proper subset** of *B*, written $A \subset B$, if $A \subseteq B$ and there is an element of *B* that is not in *A*.
Example: The set of the original thirteen colonies is a proper subset of the set of all U.S. states.
The set of letters in the word *math* is a proper subset of the set of letters in the word *mathematics*. $\{m, a, t, h\} \subset \{m, a, t, h, e, i, c, s\}$



Union of Sets: $A \cup B$

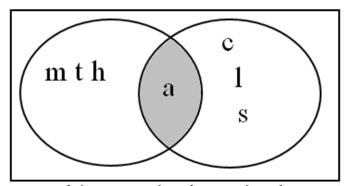
The **union** of two sets *A* and *B*, written $A \cup B$, is the set of all elements belonging to either *A* or to *B* or both.
What letters are in the word *math* or in the word *class*?
The union helps us answer that question.



If $A = \{m, a, t, h\}$, $B = \{c, l, a, s\}$, then $A \cup B = \{m, a, t, h, c, l, s\}$

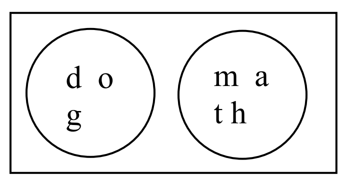
Intersection of Sets: $A \cap B$

The **intersection** of two sets *A* and *B*, written $A \cap B$, is the set of all elements common to sets *A* and *B*.
What letters are in the word *math* and in the word *class*?
The intersection helps us answer that question.



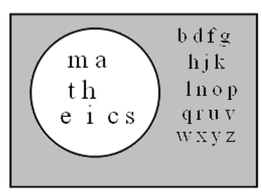
If $A = \{m, a, t, h\}$, $B = \{c, l, a, s\}$, then $A \cap B = \{a\}$

Two sets are **disjoint** if they have no elements in common.
What letters are in the word *math* and in the word *dog*?



Complement of a Set: $\overline{A}$

The **complement** of a set *A*, written $\overline{A}$, is the set of all elements in the universe that are not in *A*.
If the universe is the set of the 26 letters of our alphabet, the complement of the set of the letters in the word *mathematics* is the set of the letters of the alphabet not in the word *mathematics*.

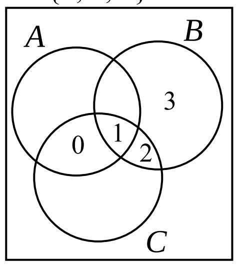


Cartesian Product of a Set:

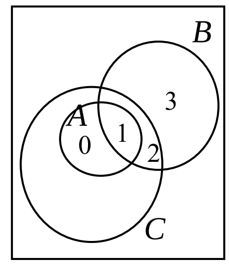
The Cartesian Product (named after Rene Descartes) of two sets *A* and *B* is the set of ordered pairs which results from matching every element of *A* with every element of *B*.
Example: If $A = \{m, a, t, h\}$ and $B = \{c, l, a, s\}$, write the Cartesian Product of *A* and *B*.

Exercises:

Suppose $A = \{0, 1\}$ $B = \{1, 2, 3\}$, $C = \{0, 1, 2\}$. Sketch a Venn Diagram to illustrate their relationship.
Here is one possible Venn diagram:



We can resize the circles to show that *A* is entirely contained in *C* and that 2 is a member of both *B* and *C* but not in *A*, and 3 is a member only to *B*. So this Venn Diagram is also correct:



Insert the appropriate symbol $\in$, $\notin$, $\subseteq$, $\subset$, or $\not\subseteq$ in the blank to make a true statement.

2 _____ B	A _____ B	C _____ $A \cup C$
2 _____ C	A _____ C	A _____ $A \cap C$

$2 \in B$
$2 \notin C$

$A \not\subseteq B$. *A* is not a subset of *B* since 0 is in *A* but 0 is not a member of *B*.

$A \subset C$. *A* is a proper subset of C since all of *A* is contained in *C*, and 2 is a member of *C* that is not in *A*.

$C \subseteq A \cup C$ since $A \cup C = \{0, 1, 2\}$ and $C = \{0, 1, 2\}$

$A \subseteq A \cap C$ since $A \cap C = \{0, 1\}$ and $A = \{0, 1\}$

Closure Property for Addition:

A set of numbers is "closed under addition" if the sum of any two members (*including themselves*) is also in that set.
$A = \{0, 1\}$ $B = \{1, 2, 3\}$, $C = \{0, 1, 2\}$.

Which of the sets *A*, *B*, and *C*, are closed under addition?
Not *A*: 1+1=2 and $2 \notin A$.
Not *B:* 2+3=5 and $5 \notin B$.
Not *C*: 1+2=3 and $3 \notin C$.

Similarly, which are closed under subtraction?
Not *A*: 0−1=−1 and $-1 \notin A$.
Not *B*: 2−2=0 and $0 \notin$ B.
Not *C*: 0−2=−2 and $-2 \notin C$.

Which of the sets are closed under multiplication?
Just set A.
Not *B* or *C* ($2 \times 2 = 4$ and $4 \notin B$ and $4 \notin$ C.

Which of the sets are closed under division?
Not *A* or *C*. $1 \div 0$ is undefined and in neither set.
Not *B*: $1 \div 2 = ½$ and $½ \notin B$.