

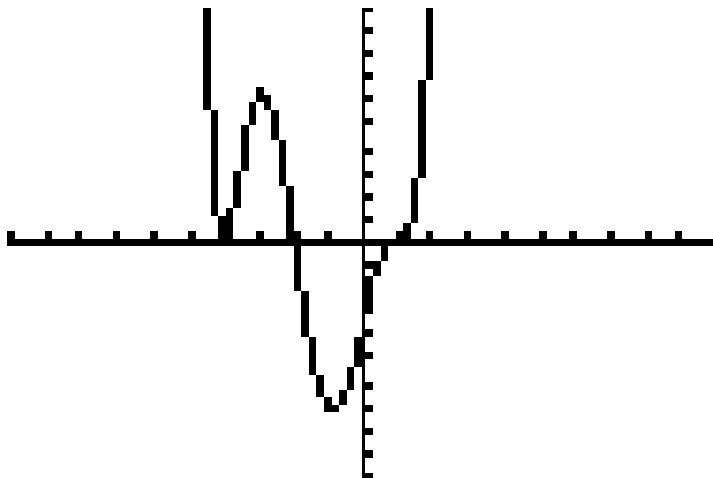
The Short Run Behavior of Polynomial Functions

In a standard viewing window $-10 \leq x \leq 10$ by $-10 \leq y \leq 10$ sketch a graph of the polynomial $p(x) = 0.1(x+4)^2(x-1)^3(x+2)$. We want to explore the shape of the graph near its zeros.

Notice we can factor $p(x)$ as $p(x) = (x+4)^2 \cdot (\text{something else})$ or

$$p(x) = (x-1)^3 \cdot (\text{something else}) \text{ or}$$

$$p(x) = (x+2) \cdot (\text{something else}). \text{ This will be useful later.}$$



1. What is the zero that results from ...

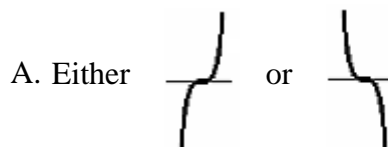
- ... the factor $(x+4)^2$? _____ What is the degree of $(x+4)^2$? _____
- ... the factor $(x-1)^3$? _____ What is the degree of $(x-1)^3$? _____
- ... the factor $(x+2)$? _____ What is the degree of $(x+2)$? _____

2. Choose from the shapes shown.

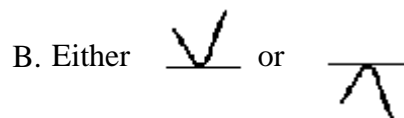
a. Which best describes the shape of

$$p(x) = 0.1(x+4)^2(x-1)^3(x+2)$$

near values of x close to $x = -4$? Choice ____.



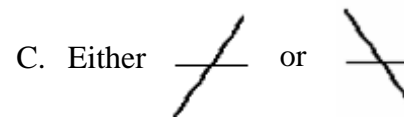
What type of graph is this? _____
(choose from *linear*, *parabola*, *cubic*)



b. Which best describes the shape of

$$p(x) = 0.1(x+4)^2(x-1)^3(x+2)$$

near values of x close to $x = 1$? Choice ____.



What type of graph is this? _____
(choose from *linear*, *parabola*, *cubic*)

c. Which best describes the shape of

$$p(x) = 0.1(x+4)^2(x-1)^3(x+2)$$

near values of x close to $x = -2$? Choice ____.

What type of graph is this? _____
(choose from *linear*, *parabola*, *cubic*)

3. Complete the blanks. The graph of $p(x) = 0.1(x+4)^2(x-1)^3(x+2)$ looks like the graph of $y = k \cdot (x+4)^2$ near values of x close to -4 , for some value of k .

a. Looking at the graph of $p(x)$, do you expect k to be positive or negative? _____
Why?

b. Enter the equation $y = (x+4)^2 \cdot 1$ in a grapher using bold style as shown.

The graph is close in shape but not quite the same:



$$y = k(x+4)^2, k = 1$$

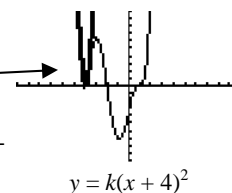
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Plot1 Plot2 Plot3
Y1=1(X+4)^2(X-1)
Y3(X+2)
Y2(X+4)^2*1

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Your target would be to find a number k so that the graph of $y = k(x+4)^2$ looks like the figure shown to the right.

Do you expect k to be greater than 1 or smaller than 1? _____
Why?



$$y = k(x+4)^2$$

c. Work together in your group to find a good value of k which makes the graph of $y = k \cdot (x+4)^2$ appear like the graph of $p(x)$ for values of x that are very close to -4 by changing the “1” in the equation $y = (x+4)^2 \cdot 1$ to some appropriate value. What value of k did your group find?
 $k =$ _____

d. At the beginning of this handout, you read that $p(x) = \frac{1}{10}(x+4)^2(x-1)^3(x+2)$ can be written as $p(x) = (x+4)^2 \cdot (\text{something else})$.
What is the “something else”?

e. For your answer to part d, replace the value of x with -4 and find the value. _____
How does it compare to your answer to part c?
Why?

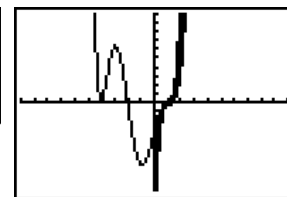
4. The graph of $p(x) = \frac{1}{10}(x+4)^2(x-1)^3(x+2)$ looks like the graph of $y = k \cdot (x-1)^3$ near values of x close to 1, for some value of k . Following what you did in Question 3, what value of k would make $y = k \cdot (x-1)^3$ look like $p(x)$ near values of x very close to 1? $k =$ _____

Enter this equation into Y2 and compare the graphs:
(Sorry, the value of k was covered by a printing smear. ☹)

```

Plot1 Plot2 Plot3
Y1=1(X+4)^2(X-1)
Y3(X+2)
Y2(X-1)^3*

```



5. The graph of $p(x) = 0.1(x+4)^2(x-1)^3(x+2)$ looks like the graph of $y = k \cdot (x+2)$ near values of x close to -2 , for some value of k . Following what you did previously, what value of k would make $y = k \cdot (x+2)$ look like $p(x)$ near values of x very close to -2 ? $k =$ _____

Do you expect k to be positive or negative? _____

Enter this equation into Y2 and compare the graphs:
(Sorry, the value of k was again covered by another smear. ☹)

```

Plot1 Plot2 Plot3
Y1=1(X+4)^2(X-1)
Y3(X+2)
Y2(X+2)*

```

