Answers to Even Numbered Problems Selected from Chapter 4 Review

Chapter 4 Review -- 1, 2 through 21, 26, 27, 30, 31 (annual growth rate only), 32i, iii, 34, 35, 36

2. (a) Accion's interest rate = (1160 −1100)/1000 = 0.16 =16%
   (b) Payment to loan shark = 1000 +(22%)•(1000) = $1220
   (c) The one from Accion, since the interest rate is lower.

4. (a) ii.
   (b) i.
   (c) iv
   (d) ii.
   (e) iii.
   (f) i.

6. $f(x) = 3\left(\frac{1}{9}\right)^x$

8. $y = 4\left(0.1\right)^x$

10. $y = \frac{1}{2}\left(\frac{1}{3}\right)^x$

12.

$$
\begin{aligned}
13e^{0.081t} &= 25e^{0.03t} \\
e^{0.081t} &= \tfrac{25}{13}e^{0.032t} \\
\tfrac{e^{0.081t}}{e^{0.032t}} &= \tfrac{25}{13} \\
e^{0.081t-0.032t} &= \tfrac{25}{13} \\
e^{0.081t-0.032t} &= \tfrac{25}{13} \\
e^{0.049t} &= \tfrac{25}{13} \\
\ln e^{0.049t} &= \ln \tfrac{25}{13} \\
0.049t &= \ln \tfrac{25}{13} \\
t &= \frac{\ln(25/13)}{0.049} \approx 13.35
\end{aligned}
$$

Another method:

$$
\begin{aligned}
13e^{0.081t} &= 25e^{0.03t} \\
\ln 13e^{0.081t} &= \ln 25e^{0.032t} \\
\ln 13 + \ln e^{0.081t} &= \ln 25 + \ln e^{0.032t} \\
\ln 13 + 0.081t &= \ln 25 + 0.032t \\
0.081t - 0.032t &= \ln 25 - \ln 13 \\
0.049t &= \ln 25 - \ln 13 \\
t &= \frac{\ln 25 - \ln 13}{0.049} \approx 13.35
\end{aligned}
$$

14. $\log 100^{x+1} = \log(10^2)^{x+1} = \log 10^{2(x+1)} = 2(x+1)$

16.

$$
\begin{aligned}
\ln(A+B) - \ln(A^{-1}+B^{-1}) &= \ln(A+B) - \ln(\tfrac{1}{A}+\tfrac{1}{B}) \\
&= \ln(A+B) - \ln(\tfrac{1}{A}\tfrac{B}{B}+\tfrac{1}{B}\tfrac{A}{A}) \\
&= \ln(A+B) - \ln(\tfrac{A+B}{AB}) \\
&= \ln(A+B) + \ln(\tfrac{A+B}{AB})^{-1} \\
&= \ln(A+B) + \ln(\tfrac{AB}{A+B}) \\
&= \ln(\tfrac{A+B}{1}\cdot\tfrac{AB}{A+B}) \\
&= \ln AB
\end{aligned}
$$

18. (a) $Q(t)$ is linear since the rate of change is very close to constant, namely 0.17
$Q(t) = 7 + 0.17t$

(b) $R(t)$ is not linear since the rate of change is not constant.
Taking ratios of successive outputs, we see these ratios are constant, namely 1.03, so $R(t)$ is exponential.
$R(t) = 2(1.03)^t$

(c) $S(t)$ is neither linear nor exponential, testing the rates of change as above.

20. (a) Assuming linear growth at 250 per year, the population in 1990 would be 18,500 + 250(10) = 21,000.
Assuming exponential growth at a constant percent rate, the percent rate would be $\frac{250}{18{,}500} \approx 0.0135 = 1.35\%$, so after 10 years the population would be $18{,}500(1.0135)^{10} \approx 21{,}155$. The town's growth is poorly modeled by both linear and exponential functions.

(b) Not possible. We do not have enough information.

26. (a) i. $0.4 = \frac{2}{5}$, so $\log \frac{2}{5} = \log 2 - \log 5 = u - v$

ii. $\log 0.25 = \log \frac{1}{4} = \log \frac{1}{2^2} = \log 2^{-2} = -2\log 2 = -2u$

iii. $\log 4\ = \log(8 \cdot 5) = \log(2^3 \cdot 5) = \log 2^3 + \log 5 = 3\log 2 + \log 5 = 3u + v$

iv. $\log \sqrt{10} = \log 10^{1/2} = \log(2 \cdot 5)^{1/2} = \frac{1}{2}\log(2 \cdot 5) = \frac{1}{2}(\log 2 + \log 5) = \frac{1}{2}(u + v)$

(b)
$$\begin{aligned}\tfrac{1}{2}(u + 2v) &= \tfrac{1}{2}(\log 2 + 2\log 5)\\ &= \tfrac{1}{2}(\log 2 + \log 5^2)\\ &= \tfrac{1}{2}\log(2 \cdot 5^2)\\ &= \log(\sqrt{50})\\ &\approx \log(\sqrt{49}) = \log 7\end{aligned}$$

30. (a) M_0 = the average man's bone mass at age 30. We have the following pattern:

a	M
30	M_0
31	$0.98\,M_0$
32	$0.98^2\,M_0$
33	$0.98^3\,M_0$
$\vdots$	$\vdots$
50	$0.98^{20}\,M_0$
70	$0.98^{40}\,M_0$

If we continue the pattern, the formula appears to be $M(a) = (0.98)^{a-30} M_0$

(b) $30 + \frac{\log 0.5}{\log 0.98} \approx 64.3$ years of age.

32.(i) Equation b (iii) Equation c

34 (a) Solve $1.06 = e^k$

$k = \ln 1.06 \quad 0.0583$. This means that $5000(1.06)^t \approx 5000e^{0.0583t}$ or an account which pays 6% compounded annually has the about same yield as an account which pays 5.83% compounded continuously.

(b) Find $b = e^{0.072} \approx 1.0747$.

This means that $7500e^{0.072t} \approx 7500(1.0747)^t$ or an account which pays 7.2% compounded continuously has about the same yield as an account which pays 7.47 % compounded annually.

36. (a) 10.41% since

$$\begin{aligned} Q &= Q_0(2)^{t/7} \\ &= Q_0 b^t \text{ where } b = (2)^{1/7} \approx 1.1041 \text{ so} \\ &= Q_0 1.1041^t \end{aligned}$$

(b) 10.5%

$$\begin{aligned} Q &= Q_0(3)^{t/11} \\ &= Q_0 b^t \text{ where } b = (3)^{1/11} \approx 1.105 \text{ so} \\ &= Q_0 1.105^t \end{aligned}$$

(c) 42.58%

$$\begin{aligned} Q &= Q_0(1.03)^{12t} \\ &= Q_0 b^t \text{ where } b = (1.03)^{12} \approx 1.4258 \text{ so} \\ &= Q_0 1.4258^t \end{aligned}$$

(d) 48.77%

$$\begin{aligned} Q &= Q_0(1.18)^{12t/5} \\ &= Q_0 b^t \text{ where } b = (1.18)^{12/5} \approx 1.4877 \text{ so} \\ &= Q_0 1.4877^t \end{aligned}$$