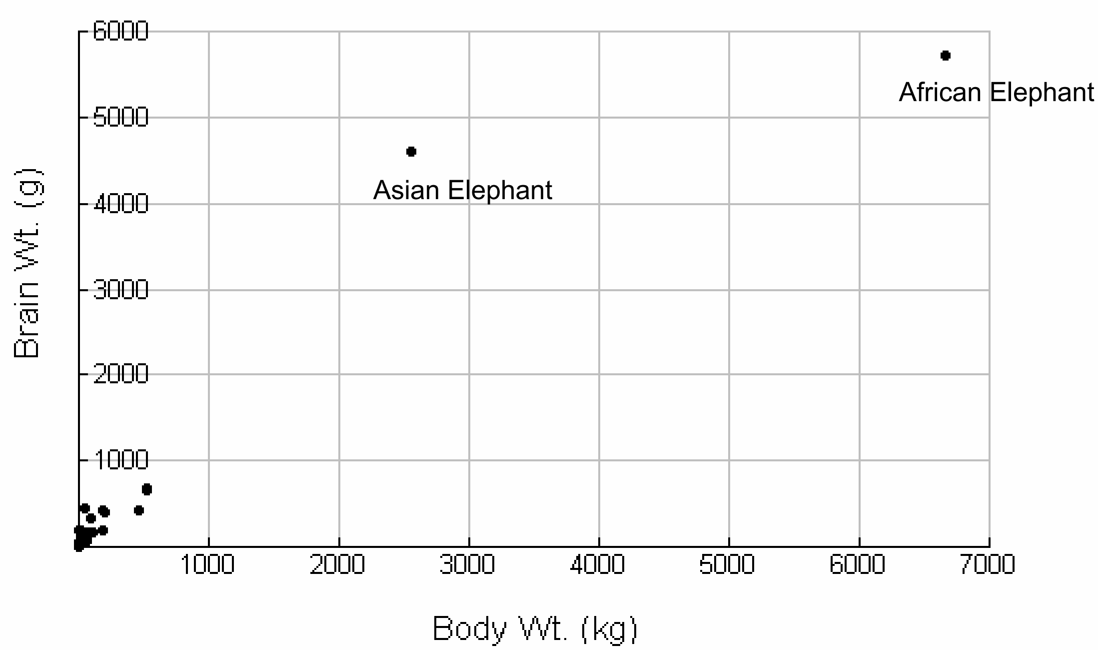
WR 9 My Friend, the Logarithm Name___________________________

Due Friday, December 8, 2006

The table and graph below give species, body weight, and brain weight for various mammals[1].

Species	**Body Wt. (kg)**	**Brain Wt. (g)**	**Species**	**Body Wt. (kg)**	**Brain Wt. (g)**
African Elephant	6654.0	5712.0	Guinea Pig	1.04	5.5
Asian Elephant	2547.0	4603.0	Horse	521.0	655.0
Baboon	10.55	179.5	Jaguar	100.0	157.0
Big Brown Bat	0.023	0.3	Kangaroo	35.0	56.0
Cat	3.30	25.6	Mouse	0.023	0.4
Chimpanzee	52.16	440.0	Opossum	1.70	6.3
Cow	465.0	423.0	Pig	192.0	180.0
Donkey	187.1	419.0	Rabbit	2.50	12.1
East. American Mole	0.075	1.2	Raccoon	4.29	39.2
Giant Armadillo	60.0	81.0	Rat	0.28	1.9
Giraffe	529.0	680.0	Red Fox	4.24	50.4
Goat	27.66	115.0	Rhesus Monkey	6.80	179.0
Golden Hamster	0.12	1.0	Roe Deer	14.83	98.2
Gorilla	207.0	406.0	Sheep	55.50	175.0
Gray Seal	85.0	325.0	Tree Shrew	0.104	2.50
Gray Wolf	36.33	119.5	Yellow-bellied Marmot	4.05	17.0
Ground Squirrel	0.10	4.0			



[1]In the article "Sleep in Mammals, Ecological and Constitutional Correlates", by Truett Allison and Domenic Cicchetti, *Science*, Volume 194, Issue 4266, Nov. 12, 1976, pp. 732-734, the interrelationship between sleep, brain weight, body weight, and other constitutional variables and ecological factors were studied. A representative data set comparable to this one is provided.

Think about the following questions in your group.

a. Is there is any relationship between body size and brain size?
b. If so, what kind of relationship exists? Does it appear linear?
c. What assumptions must be made for our model to be valid?
d. What is the size of a human brain?

The goal of this investigation is to see how mathematics can help answer the above questions.

I. Preliminaries: Bodies and Brains

We can link the (*body weight, brain weight*) data given on the previous page directly to our graphers and conduct a preliminary investigation. The data is stored in the lists ÙBODY and ÙBRAIN. The calculator program BB.8XP puts these lists into your Stat Editor and also produces the plot.

1. Once you have BB.8XP on your TI-83 Plus, run the program. Press s .
   Run the command DiagnosticOn found in the catalog and then find a **linear** model of the data.
   What assumptions were used?
   Report the correlation coefficient, *r*: ____________

2. Suppose you wanted to use your model to determine the size of your brain, in grams, given your weight in kilograms. For fun, do this now (1 lb ≈ 0.454 kg.) and report it:
   ________ g of brain mass.

II. Using Balls to Model Bodies and Brains

Paleontologist Stephen Jay Gould thought about the biological context of the situation when trying to conjecture what kind of relationship there might be between brain size and body size.
He used three simplifying assumptions:

1. All body tissue has roughly the same density,
   so volume and mass are proportional.

2. Brain size is roughly proportional to the number of external stimuli,
   which is in turn roughly proportional to the surface area of the body.

3. All animals are spherical.

From these assumptions Gould proposed that brain mass and body mass are related in the same way that the surface area *SA* of a sphere is related to its volume *V*. We now explore the latter.

1. Your group will need a *Reader, Recorder, Quality Controller,* and *Reporter.*
   Without taking any measurements, make a reasonable estimate of the following quantities for your ball:

   a. Check which item you think is **larger**:
      ___ the number of cubic centimeters which could be used to fill your ball
      (Imagine constructing a model of your ball out of 1 cm $\times$ 1 cm $\times$ 1cm sugar cubes and whittling away the edges to make a sphere, counting every portion of sugar cube that remains.)

      ___ the number of square centimeters which could be used to cover your ball
      (Imagine wrapping your ball with paper that is marked with 1 cm $\times$ 1 cm grids.)

   b. Now record your estimate of the number of cubic centimeters needed to fill your ball: _________

   c. Now record your estimate of the number of square centimeters of wrapping paper needed to cover the outside of your ball: _________

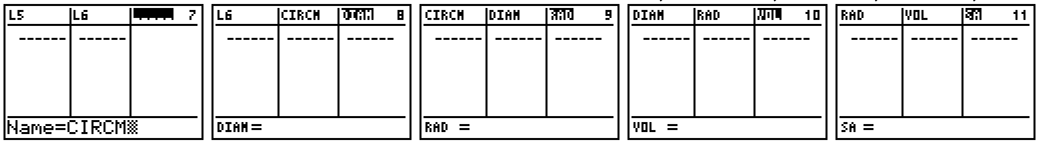
2. Discuss in your group the best way to determine the volume and surface area of your ball. You will need the formulas $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$. Make measurements and calculations and report your answers using appropriate metric units.

   Ball # ______ Volume: _____________ Surface Area: _______________

   Explain your procedure:

3. Use the space below to record the data that each group collected.
   We will now enter the data in our graphing calculator.

4. Use the Stat Editor to create the five lists ÙCIRCM, ÙDIAM, ÙRAD, ÙVOL, and ÙSA.

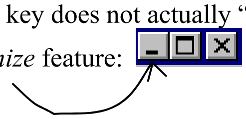


Enter the data recorded in Step 3 in just the list ÙCIRCM.
(Data for the remaining lists will be calculated in the next step.)

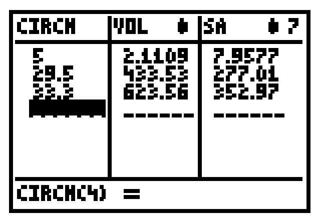
5. Use formulas (**Important**: with *quotes*!) to calculate the values for the four remaining lists ÙDIAM, ÙRAD, ÙVOL, and ÙSA.

6. **Check this now:** Perform a quality control at this point.
If done correctly, you should see the ◆ symbol next to the names of the four lists ÙDIAM, ÙRAD, ÙVOL, and ÙSA in the Stat Editor. If not done correctly, repeat Step 5 before continuing, this time using quotes at the beginning of the formula.

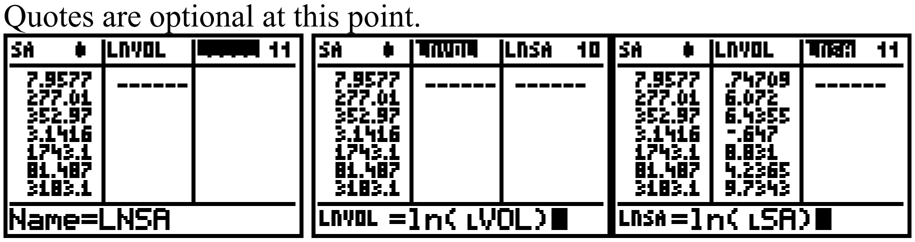
___ Check here if you pass inspection.

7. In the Stat Editor use the { key to remove the lists ÙDIAM, ÙRAD so that the three lists ÙCIRCM, ÙVOL, and ÙSA are side by side.
(In the Stat Editor, the { key does not actually "delete" the lists from memory, but behaves similar to Microsoft Windows' *minimize* feature: )

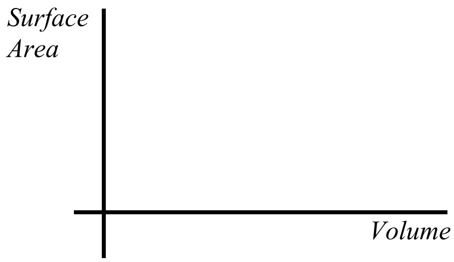
Check that your screen looks similar to this:



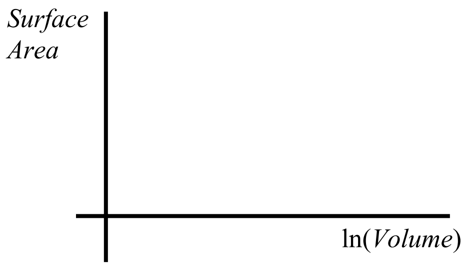
8. Scientists often look for patterns in relationships by re-expressing the data using logarithms. Create two more lists ÙLNVOL and ÙLNSA in the Stat Editor and define them ÙLNVOL = ln(ÙVOL) and ÙLNSA = ln(ÙSA) as shown.
Quotes are optional at this point.



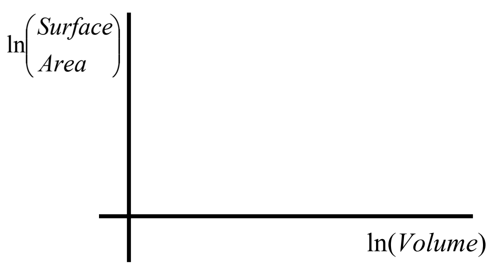
Divide the work among members of your group to create each plot below.
Make a rough copy of your plots on the axes below. Which model is the most linear?
Find **linear** models and report the correlation coefficient *r* for each.



$r =$ ______________

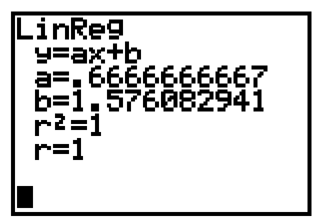


$r =$ ______________



$r =$ ______________

9. Since the outputs were *calculated* from the inputs as opposed to being measured, one of the plots in the previous Question will be perfectly linear. Notice one of these numbers in the model's equation can easily be recognized. What is its exact value? ___ (The other one is a numerical expression involving π.)

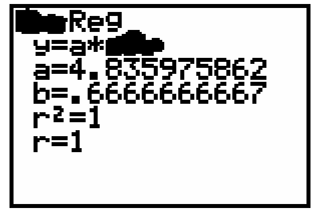


a. When $x = \ln V$ and $y = \ln SA$, you have a linear model $y = ax + b$ which relates x and y. By substituting 1.576 for b and the appropriate exact value for a, we can write the linear model relating $\ln V$ and $y = \ln SA$ by completing the blank:

$$\ln SA = \_\_\_\_\_ \ln V + 1.576$$

b. Manipulate the equation in part a to find a relationship between SA and V which doesn't involve logarithms. Your new model will either be a power function or an exponential function.

c. Run the appropriate command (PwrReg ÙVOL, ÙSA or ExpReg ÙVOL, ÙSA) to confirm your work in part b. Your screen should look like the one to the left (without the ink smudges!)



d. A theoretical approach to finding the relationship between *SA* and *V* is to use the formulas $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$. Write *r* as a function of *V*, and substitute this into the formula for *SA*. Then use algebra to find the exact value of $a \approx 4.835975862$ (Your answer will be a numerical expression involving π.)

i. Write r as a function of *V*: $r(V) =$

ii. Substitute your expression $r(V)$ into the formula $SA = 4\pi r^2$ so that *SA* is in terms of *V*. Simplify so that it is a power function.

iii. What is the **exact** value of $a \approx 4.835975862$? ________________

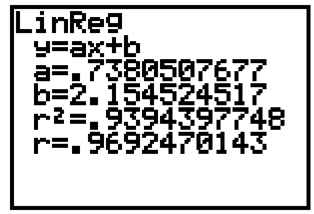
III. Balls, Bodies and Brains

Paleontologist Stephen Jay Gould's simplifying assumptions led him to conjecture that brain mass and body mass are related in the same way that the surface area *SA* of a sphere is related to its volume *V*. Therefore, brain mass is proportional to body mass raised to the $\frac{2}{3}$ power.

1. Rerun the program BB to restore the (*body weight, brain weight*) plot and clean up the Stat Editor. Since the plot of (ln *V*, ln *SA*) was perfectly linear, we will investigate if the plot of (ln *body weight,* ln *brain weight*) has a linear relationship.

2. In the Stat Editor create two more lists ÙLNBOD and ÙLNBRN and define them ÙLNBOD = ln(ÙBODY) and ÙLNBRN = ln(ÙBRAIN).

3. Make a plot of (ln *body weight,* ln *brain weight*). Turn off any other plots.
   Does the plot appear linear? _______
   Graph the linear regression model.
   The screen to the right shows the diagnostics for the model
   ln *brain weight* ≈ 0.738 ln *body weight* + 2.154



4. Rerun the program BB again to restore the (*body weight, brain weight*) plot.
   Graph the power regression model.
   Study the diagnostics screen for the power regression. Recopy these values:
   a = _________________________
   b = _________________________
   r = _________________________
   Discuss with a partner how the values compares with the above diagnostics screen for the linear regression shown in the previous Question. What is the same? _____________
   What is different? ____
   Why? How are the values that are different related?

5. For fun, use your power model to determine the size of your brain, in grams, given your weight in kilograms (1 lb ≈ 0.454 kg.) and report it:

   ________ g of brain mass.

   Now convert this back to pounds: __________ lb of brain

   Does it make sense to use the model this way? Explain.

IV. More with Brains

t, min	*P* (%)
5	73.0
15	61.7
30	58.3
60	55.7
120	50.3
240	46.7
480	40.3
720	38.3
1440	29.0
2880	24.0
5760	18.7
10080	10.3

A psychologist conducts the following experiment to examine how the brain remembers. A person (subject) read a list of 20 words slowly aloud. Later the subject was shown a list of 40 words which contained the original 20, and was asked which words he or she recognized. The percent, *P*, was calculated. This was repeated at various time intervals.

The average percentages of five subjects in the study are shown at various time intervals in the table to the right.
Adapted from D. Lewis, *Quantitative Methods in Psychology*, (New York: McGraw-Hill, 1960).

1. Construct a plot of the data. Notice time is given in minutes. What value should Xcsl be to mark the *x*-axis in days? _______ Set Xscl to this value. How many days was this study conducted? ____

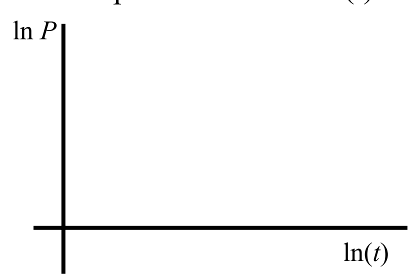
Why the plot look the way it does? Be specific. Does this match your experience with memory? (You might talk about what happens to *P* and the rate of change of *P* as *t* increases.)

2. You have seen how scientists make finding a model less of a mystery: they re-expressing the data using logarithms and hope they see a more recognizable function model. Create the three plots below and recopy your screens. Perform a linear regression on each and recopy *r*.

I. A plot of ln *P* against *t*

r = ____________________

II. A plot of ln *P* vs. ln (*t*)



r = ____________________

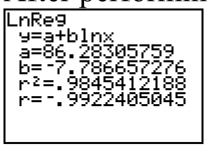
III. A plot of P vs. ln (*t*)

r = ____________________

3. Which of these plots looks linear? (Circle one) **I** **II** **III**
   Using the coefficients of your linear regression, rounded to three decimal places, write an equation using variables P, t, $\ln(P)$, or $\ln(t)$. (Do not write an equation involving x and y.)

   ____

4. Return to the original plot of P against t and include the equation you wrote above in Y1.
   After performing a natural log regression on the (t, P) data, a student sees the following screen



   How does the value of r compare to what you found in Question 2? Explain what is happening.

5. The psychiatrist is interested in the following question below. You are hired as a lab assistant. Use your equation.

   *When does the model predict that the subject will first recognize **no** words?*
   (Round to the nearest 0.1 day.)