MA 153 Writing Assignment 4 Compounding and *e* Name_____

Due Tuesday 10-21-08 (10 pts)

(1) 1. In the Spring of 1990, the New York Times reported each of the articles below.

A fool and his money are ***soon*** parted! How wise to wait ***200 years*** to part with it while the interest is compounded!

a. Reported in the New York Times, April 21, 1990.

Benjamin Franklin bequeathed some money to the cities of Boston and Philadelphia with the condition that it could not be spent for 200 years. He died in 1790 and the money became available in 1990.

The amount that Franklin's gift had become was approximately equivalent to the result of putting $2550 into a bank account drawing 4% annual compound interest for 200 years.

Old Ben was pretty smart! Determine the amount of money his bequest had become in 1990. (Since, banks set a limit to the length of time you can compound the interest of an investment.)

Amount = _____

(4) b. Reported in the New York Times, May 27, 1990 (one month later). Complete the boxes.

213 Years After Loan, Uncle Sam is Dunned

By LISA BELKIN

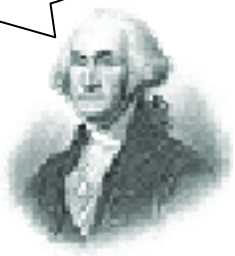
SAN ANTONIO, May 26 – More than 200 years ago, a wealthy Pennsylvania merchant named Jacob DeHaven lent $450,000 to the Continental Congress to rescue the troops at Valley Forge. That loan was apparently never repaid.

So Mr. DeHaven's descendents are taking the United States Government to court to collect what they believe they are owed. The total: [ ] in today's dollars if the interest is compounded daily at 6 percent, the going rate at the time. If compounded yearly the bill is only [ ].

Family is Flexible

The descendents say that they might even accept a heartfelt thank you or perhaps a DeHaven statue. But they also note that interest is accumulating at [ ] a second.

I cannot tell a lie. Jacob told me personally it was a GIFT. Yeah, that's it.



Assume the interest has been compounded yearly.

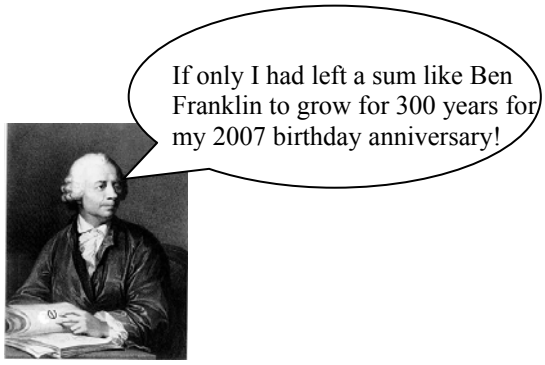
Hint:

First, find [*Total Amount in 214 Years*] – [*Total Amount in 213 Years*] = $ _____ /year

Then convert this amount to dollars of interest per second = $ _____ /sec

2. Leonhard Euler* gave us the number *e*, which is what the expression $(1+\frac{1}{x})^x$ approaches as *x* gets larger and larger.
For example, suppose $x = 10{,}000$ or 10^4.
Then $(1+\frac{1}{x})^x = (1+0.0001)^{10000} = 1.0001^{10000} = 2.71814592...$

*See www.usna.edu/Users/math/meh/euler.html for more on Lenny E.
On April 15, 2007 we celebrated the 300th anniversary of his birth.



(1) a. Complete the last four rows of the table below:

x	$y = (1+\frac{1}{x})^x$ (Numerical Expression)	Decimal Approximation
10^4	$1.0001^{10,000}$	2.71814592683…
10^5	$1.00001^{100,000}$	2.71826823717…
10^6	$1.000001^{1,000,000}$	2.71828046932…
10^7		
10^8		
10^9		
10^{10}		

Graph $y = (1+\frac{1}{x})^x$ on your calculator, using a window from $0 \le x \le 10$,
and then for $0 \le x \le 100$, and then $0 \le x \le 1000$, and then for $0 \le x \le 10000$.
As values of *x* get larger and larger, complete the boxes to describe what the graphs and above table suggest about the function $y = (1+\frac{1}{x})^x$: as $x \to$ ☐, $y = (1+\frac{1}{x})^x \to$ ☐.

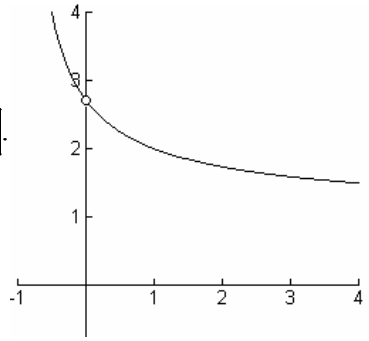
We call the line $y =$ ☐ a **horizontal asymptote** of the graph of $y = (1+\frac{1}{x})^x$.

(1) b. Another approach is to examine the function $(1+x)^{1/x}$. Complete the table:

x	$y = (1+x)^{1/x}$ (Numerical Expression)	Decimal Approximation
10^{-4}		
10^{-5}		
10^{-6}		
10^{-7}		
10^{-8}		
10^{-9}		
10^{-10}		

Describe any similarities with this table and the table in part (a).

Reproduce the graph of $y = (1+x)^{1/x}$ shown to the left on your calculator.
What is the value of the function at $x = 0$? _____
From the table and graph, we can write that as $x \to$ ☐, $y = (1+x)^{1/x} \to$ ☐.
We call the point $(0,$ ☐$)$ a **hole-in the graph** of $y = (1+x)^{1/x}$.



c. Notice the pattern $1.0001^{10,000}$, $1.00001^{100,000}$, $1.000001^{1,000,000}$, … which continues indefinitely. For example, $1.00000000000000000000000000001^{100,000,000,000,000,000,000,000,000,000} \approx e$.
Two competing properties influence these expressions:

(1) i. What is true about the power of 1? $1^{10,000}$, $1^{100,000}$, $1^{1,000,000}$, … , $1^{100,000,000,000,000,000,000,000,000,000}$

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(1) ii. If a number n is greater than 1, what happens as you raise it to larger and larger powers? For example, $n^{10,000}$, $n^{100,000}$, $n^{1,000,000}$, … , $n^{100,000,000,000,000,000,000,000,000,000}$, …

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The number e can be said to be a tradeoff of these two properties.

(1) 3. In the space below, (it need not be typed), summarize what you learned about the functions function $y = (1+\frac{1}{x})^x$ and $y = (1+x)^{1/x}$ and the advantages each of them has in approximating the number e. Hint: $y = (1+x)^{1/x}$ has a real-life interpretation that was mentioned in class. For $y = (1+x)^{1/x}$, look at Question 2b and 2c.