Additional Tips for Review

Section 5.1 – This is a pretty important section, and is used throughout the remaining chapters. I may give you a graph of a shift of $f(x) = \ln x$ or an exponential function and ask you to find its formula. Once you come up with the formula, you might be asked follow-up questions like: what is the domain or what is the y-intercept or x-intercept.

For $c > 0$, remember for vertical shifts that

$f(x)+c$ shifts $f(x)$ **up** c units, while

$f(x)-c$ is a shift of $f(x)$ **down** c units.

However, horizontal shifts are trickier and counterintuitive.

<u>Remember this rhino tip!</u>

Sketch $f(x+c)$ or $f(x-c)$ according to the following:

> What happens to the graph of $f(x+c)$ at the value $x = -c$ is exactly what happens to the graph of $f(x)$ at the value $x = 0$.

Similarly,

> What happens to the graph of $f(x-c)$ at the value $x = c$ is exactly what happens to the graph of $f(x)$ at the value $x = 0$.

This will never steer you wrong, and you won't get messed up by shifting the wrong way

You can check points other than $x = 0$ as well, but 0 usually is the easiest.

Section 5.2 – Horizontal and vertical reflections are added to the tool box.

A negative on the *inside*, $f(-x)$, is a *horizontal* reflection of $f(x)$, and on the *outside*, $-f(x)$, is a *vertical* reflection. A change to the input is a horizontal change; a change to the output is a vertical change. Notice the pattern was the same for shifts too. Questions like **11-17** are good.

You must know what it means to be an even function or an odd function by inspection.

The blue boxes at the bottom of pages 193 and 194 summarize this. Problems like **18-21** should be a snap, especially if you are given a graph. A strategy for problems like **28** and **30** is to think of its graph. Problems **28** and **29** are similar.

Section 5.3 – Vertical stretches or compressions are important, and they are used in such later sections as **5.5**, **9.1** and **9.4**. In fact, look at Exercise 10 on page 205 and see how it ties in with section **5.5**. Problems like **7**, **8** and **23-25**

Section 5.5 –This section is difficult often because students don't know where to begin.

If that's the case, you just haven't done enough problems. So get to work!

If I give you a problem like **8** on page 219, you better be using factored form or you will be spinning your wheels. Your first step is to write $y = a(x+4)(x-5)$. Don't assume a is just 1 or -1.

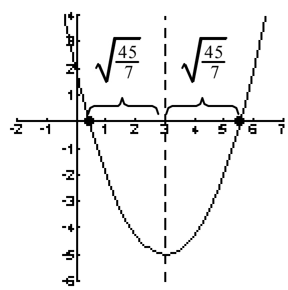
To find *a,* you plug in $x = 2, y = 36$ and solve $36 = a(6)(-3)$, which makes $a = -2$. That's not hard.

It's done again in **9.2** and **9.4**. It's worth it to understand this strategy.

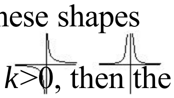
Now back to problem **8**: Notice you expected a to be a negative number. Now report $y = -2(x+4)(x-5)$ and be done with it. Don't waste time expanding it into standard form. One issue you will face on this test is using your time wisely. It's worth it, however, to check it with your grapher to save you from silly mistakes, like solving $36 = a(6)(-3)$ incorrectly as $a = -\frac{1}{2}$. By the way, students who set this up $y = -a(x+4)(x-5)$ usually botch it. They get $a = 2$ and report $y = 2(x+4)(x-5)$ which is wrong. If you are given a vertex and another point, such as problem **7**, you solve it differently of course: using vertex form. (Hence the name!) So, for **7**, you write $y = a(x-3)^2 - 5$ and plug in the values $x = 0, y = 2$ to find a. Solve $2 = a(0-3)^2 - 5$ which gives $2 = 9a - 5$ (move that 5 to the other side), so $7 = 9a$ or $a = \frac{7}{9}$. Then finish it off by writing $y = \frac{7}{9}(x-3)^2 - 5$.

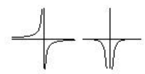
Take 30 more seconds and enter it in a grapher to check.
Parentheses are nonoptional when you enter it: Y=(7/9)(X −3)^2 − 5.
If given both the vertex as well as the x-intercept(s) like problem **14** on page 219 or **Chapter 5 Review** problems **14** and **15** on page 222, you are in luck! You can use either vertex form or factored form! Notice that standard form is hardly ever used. If you see a problem like **25**, **28**, and **29**, these factor easily since there is no constant term in the equation. Get the two zeros by setting each factor equal to 0, find the axis of symmetry midway between them, and plug the x-coordinate of the vertex into the function formula to find the y-coordinate of the vertex. I'm not too interested in having you complete the square, but you should be able to find analytically the x-intercepts and y-intercepts of anything given in vertex form. I'll ask for exact answers. For example, the zeros of the function $y = \frac{7}{9}(x-3)^2 - 5$ graphed in Problem **7** on page 219 would be $3 \pm \sqrt{5(\frac{9}{7})}$ or $3 \pm \sqrt{\frac{45}{7}}$ and, yes, you can leave it, just like that. (Aren't I a nice guy!) Know what I mean by *exact* solutions: the numbers $3 \pm \sqrt{\frac{45}{7}}$ are exact.
Reporting 0.4645372358145845 or 5.5354627641855 are hardly exact. They would not be exact if you took them out to a mere 100 decimal places. These are approximations.
Notice the slick strategy: Set $y = 0$ and solve for x, so

$$0 = \frac{7}{9}(x-3)^2 - 5$$
$$\frac{7}{9}(x-3)^2 = 5$$
$$(x-3)^2 = 5(\frac{9}{7})$$
$$x - 3 = \pm\sqrt{5(\frac{9}{7})}$$
$$x = 3 \pm \sqrt{5(\frac{9}{7})} = 3 \pm \sqrt{\frac{45}{7}}$$



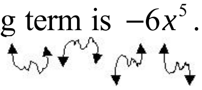
Connect this with the graphical representation of the solution to see that the zeros are exactly the same distance of $\sqrt{\frac{45}{7}}$ from the axis of symmetry, $x = 3$. You need not report intercepts as ordered pairs, but if you do, do it right. For example, if I asked you for the zeros of problem **8** on page 219, you would NOT write $(-4,5)$, but $-4,\ 5$. However the vertex of problem **7** is NOT written as $3,-5$. The text shows how these would be written, namely $(3,-5)$.

Section 8.1 – Essentially, the big idea here is that one function's output is another function's input.
Section 9.1 –This serves as a foundation for the remaining sections of this chapter. When you see $y=\frac{1}{x}$, $y=\frac{1}{x^3}$, … and $y=\frac{1}{x^2}$, $y=\frac{1}{x^4}$,… these shapes  should come to mind.
If these equations are multiplied by a constant k, $k>0$, then the shapes are basically the same, only stretched

or compressed, and if $k<0$, they're also upside down: 

Similarly, know the shape of functions in these families:
$y=x^2, y=x^4, y=x^6...$
$y=x^3, y=x^5, y=x^7,...$
$y=x^{1/2}, y=x^{1/4}, y=x^{1/6},...$
$y=x^{1/3}, y=x^{1/5}, y=x^{1/7},...$
Again, if they are multiplied by a constant k, $k>0$, then the shape is basically the same, only stretched or compressed, and if $k<0$, it's upside down, which goes back to Section 5.2 and 5.3.

You will be asked to find the formula of a power function. Using logarithms may be handy. Review all of the assigned problems from this section, like **1-13** and **18-25 odd.** Do the evens if you need more practice. You can check your answers to even problems the same way you would do so on the test, by seeing if your given numbers satisfy your formula. Try Problem **46** on page 427 out of the **Chapter 9 Review**.

Section 9.2 – Know terminology. I may give you a polynomial like $f(x)=7x^{10}-3x^{12}+4x^{15}$ or $g(x)=-6(x+30)^3(x-40)^2$ and ask for the degree and leading term. For $f(x)$ the degree is 15 (it's not x^{15} or 37) and the leading term is $4x^{15}$, but for $g(x)$ the degree is 5 (it's not x^5) and the leading term is $-6x^5$. From here you can get long run behavior. I'll ask you to specify as one of these: . Remember your work in **9.1**? It comes in quite handy here since the long run behavior of the polynomial is the same as long run behavior of its leading term, which is a power function. How convenient! The answers to exercises 9-12 should be $16x^3, 4x^4, 2x^9, 5\sqrt{x}$, respectively.

Section 9.3 – Expect a problem like **12** through **24** or **28** or **30**. This is very much like section **5.5**, only with more linear factors. Know the connection between the power of the linear factor and what happens at the zero of that factor. If the power of the linear factor is 1, the graph crosses the x-axis at the zero of that factor.
If the power on the linear factor is 3, 5, 7, … then at the zero of that factor it crosses the x-axis like $y=x^3, y=x^5, y=x^7,...$ (this is called an inflection point, but don't worry about the name.)
If the power on the linear factor is even, we have a bounce or kiss. (See how Section **9.1** is used *again.*) When finding a formula from a given graph, don't forget to include a. Your grapher can be a nice tool to check your formula. Another idea of this section is finding zeros and factoring a polynomial, such as problems **1-4**, and **34-38.** The strategy is to use the graph to help you factor.

It's most likely not the way you were taught to factor in previous courses, but you're in MA 153 now. Let go of the past.

Section 9.4 – Rational functions are ratios of polynomials. Problem **8** is nice because it brings back the concept of odd and even symmetry from **5.2**. Finding right and left hand behavior is a fancy name for finding whether or there is a horizontal asymptote. Remember, if asked for the horizontal asymptote, you must report it as an equation or it is wrong. So begin by writing "*y* =." Have you been doing this in your homework (that question assumes you ARE doing homework)? If not, get in this habit. The box at the bottom of page 398 summarizes this section really nicely! Once you look at the ratio of the leading terms, use what you know from section **9.1** to get the horizontal asymptote, if there is one. If there isn't, such as for the example $f(x)=\dfrac{6x^4+x^3+1}{-5x+2x^2}$ on page 397 and 398, simply report "NO horizontal asymptote." (If I asked what power function *f*(*x*) resembled for large values of *x*, you would report $f(x)=\dfrac{6x^4+x^3+1}{-5x+2x^2}\approx\dfrac{6x^4}{2x^2}=3x^2$)

Examples 1 and **2** on pages 398-399 are worth reading carefully.

Below is a more detailed summary. Generalizing with letters, though, gets a little messy:

Assume $f(x) = \dfrac{ax^{\text{<some power>}} + \text{some remaining terms}}{bx^{\text{<some power>}} + \text{some remaining terms}}$

- If the ratio of leading terms of $f(x)$ reduces to $y = \dfrac{ax^n}{b}$ for some positive integer *n*, then the degree of the numerator was **more than** the degree of the denominator,

  and,

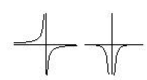
  as x gets large, $f(x)$ looks like $y = \dfrac{ax^n}{b}$, i.e.,  .

  This means there is **no** horizontal asymptote. *Example:* $y = \dfrac{2x^5 - 4x^2 + 3x}{7x^2 + 12}$ has no h.a.

- If the ratio of leading terms of $f(x)$ reduces to $y = \dfrac{a}{bx^n}$ for some positive integer *n*, then the degree of the numerator was **less than** the degree of the denominator,

  and,

  as x gets big, $f(x)$ looks like $y = \dfrac{a}{bx^n}$, i.e. 

  (or reflections of these if $\frac{a}{b}$ turns out to be negative: )

  This means the horizontal asymptote is $\boldsymbol{y = 0}$. *Example:* $y = \dfrac{7x^2 + 12}{2x^5 - 4x^2 + 3x}$ has h.a. $y = 0$

- If the ratio of leading terms of $f(x)$ reduces to $y = \dfrac{a}{b}$ for some positive integer *n*, then the degree of the numerator was **the same as** the degree of the denominator,

  and,

  as x gets big, $f(x)$ looks very much like the horizontal line $y = \dfrac{a}{b}$.

  In other words, the horizontal asymptote is $\boldsymbol{y = \dfrac{a}{b}}$. *Ex:* $y = \dfrac{7x^5 + 12}{2x^5 - 4x^2 + 3x}$ has h.a. $y = \frac{7}{2}$

Review your homework and notes, and what I emphasized above.