

Why are the logarithmic properties true?

1. Complete the blanks and boxes to show  $\log_b QR = \log_b Q + \log_b R$

- Let  $\log_b Q = x$ .
- Write the equation in 1a in exponential form: \_\_\_\_\_
- Let  $\log_b R = y$ .
- Write the equation in 1c in exponential form: \_\_\_\_\_
- $QR = b^x \cdot \square$  if we substitute the results from 1b and 1d.
- $QR = b^{\square}$  by properties of exponents. (Hint: See Rule 1 in the box on page 492 of your text.)
- Now write the equation in 1f in logarithmic form:  $(QR) = b^{\square}$  means  $\log_b \square = \square$
- Eliminate  $x$  and  $y$  in the equation in 1g by substituting the equations in 1a and 1c:  
\_\_\_\_\_

2. Complete the blanks and boxes to show  $\log_b Q^k = k \cdot \log_b Q$

- Let  $\log_b Q = x$ .
- Write the equation in 2a in exponential form: \_\_\_\_\_
- $Q^k = (\square)^k$  if we substitute 2b.
- $Q^k = b^{\square}$  by properties of exponents. (Hint: See Rule 3 in the box on page 492 of your text.)
- Now write the equation in 2d in logarithmic form:  $(Q^k) = b^{\square}$  means  $\log_b \square = \square$
- Eliminate  $x$  in the equation in 2e by substituting the equation in 2a:  
\_\_\_\_\_

3. Complete the boxes to show  $\log_b \frac{Q}{R} = \log_b Q - \log_b R$

$$\begin{aligned}
 \text{Since } \frac{1}{R} = R^{\square}, \text{ we have } \log_b \frac{Q}{R} &= \log_b (Q \cdot \square) \\
 &= \log_b (Q \cdot R^{\square}) \\
 &= \text{_____ using the property in 1h above.} \\
 &= \text{_____ using the property in 2f above.}
 \end{aligned}$$