Identities

1. $\dfrac{\cos x}{1-\sin x} - \tan x =$

   A. $\sin x$ B. $\cos x$ C. $\csc x$ D. $\sec x$ E. None of these.

2. $\dfrac{\dfrac{1}{\cos x} - 1}{1 - \cos x} =$

   A. $\sin x$ B. $\cos x$ C. $\csc x$ D. $\sec x$ E. None of these.

3. Solve on the requested interval.
   a. $2\cos^2\theta = 3\sin\theta + 3$ on $[0, 2\pi)$
   b. $2\cos\theta \sin\theta = \sin\theta$ on $[0, 2\pi)$
   c. $2\cos^2\theta - 3\cos\theta = 2$ on $(-\infty, \infty)$
   d. $\sin 2\theta = \sin\theta$ on $[0, 2\pi)$
   e. $\cos 2\theta = -\cos\theta$ on $[0, 2\pi)$

4. Write as algebraic expressions
   a. $\cos(\sin^{-1} 2x)$
   b. $\cot(\tan^{-1} 2x)$
   c. $\cos(\cos^{-1} x + \sin^{-1} x)$
   d. $\cos(2\sin^{-1} x)$

5. $\sin(x-y)\cos y + \cos(x-y)\sin y =$

   A. $\sin x$ B. $\cos x$ C. $\sin y$ D. $\cos y$ E. None of these.

ANSWERS

1. D

$$\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \frac{\cos x}{1-\sin x}\cdot\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}\cdot\frac{1-\sin x}{1-\sin x} = \frac{\cos^2 x - \sin x(1-\sin x)}{(1-\sin x)\cos x}$$

$$= \frac{\cos^2 x - \sin x + \sin^2 x}{(1-\sin x)\cos x} = \frac{\cos^2 x + \sin^2 x - \sin x}{(1-\sin x)\cos x} = \frac{(1-\sin x)}{(1-\sin x)\cos x} = \frac{1}{\cos x} = \sec x$$

2. D

$$\frac{\frac{1}{\cos x}-1}{1-\cos x}\cdot\frac{\cos x}{\cos x} = \frac{(\frac{1}{\cos x}-1)\cos x}{(1-\cos x)\cos x} = \frac{(1-\cos x)}{(1-\cos x)\cos x} = \sec x$$

3. a.

$2(1-\sin^2\theta) = 3\sin\theta + 3$

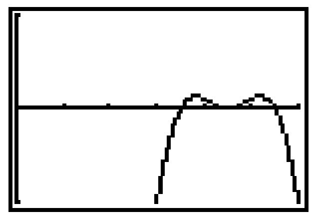
$3\sin\theta + 3 - 2 + 2\sin^2\theta = 0$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ ‖ $\sin\theta = -1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ ‖ $\frac{3\pi}{2}$



b.

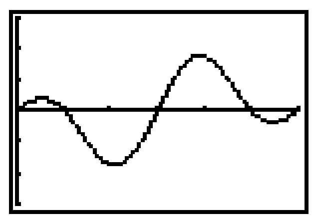
$2\cos\theta\sin\theta = \sin\theta$

$2\cos\theta\sin\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ ‖ $\cos\theta = \frac{1}{2}$

$\theta = 0, \pi$ ‖ $\frac{\pi}{3}, \frac{5\pi}{3}$



c.

$2\cos^2\theta - 3\cos\theta = 2$

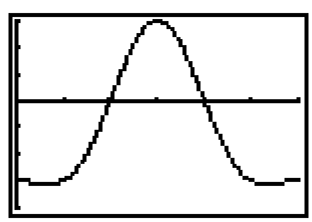
$2\cos^2\theta - 3\cos\theta - 2 = 0$

$(2\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = -\frac{1}{2}$ ‖ $\cos\theta = 2$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ ‖ None

$\theta = \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k$



d. Same as b.

e.

$\cos 2\theta = -\cos\theta$

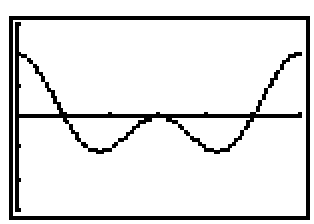
$2\cos^2\theta - 1 = -\cos\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ ‖ $\cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ ‖ π



4. a. $\cos(\sin^{-1} 2x) = \cos\theta$, where $\theta = \sin^{-1} 2x$ or $\sin\theta = 2x$

$\cos(\theta) = \sqrt{1-(\sin\theta)^2} = \sqrt{1-(2x)^2} = \boxed{\sqrt{1-4x^2}}$

b. $\cot(\tan^{-1} 2x) = \cot\theta$, where $\theta = \tan^{-1} 2x$ or $\tan\theta = 2x$

$\cot(\theta) = \dfrac{1}{\tan\theta} = \boxed{\dfrac{1}{2x}}$

c. $\cos(\cos^{-1} x + \sin^{-1} x) = \cos(\alpha + \beta)$, where $\alpha = \cos^{-1} x$, $\beta = \sin^{-1} x$

$\cos\alpha = x, \quad \sin\beta = x$

$\sin\alpha = \sqrt{1-x^2}, \quad \cos\beta = \sqrt{1-x^2}$

$$\begin{aligned}\cos(\alpha+\beta) &= \cos\alpha \cos\beta - \sin\alpha \sin\beta \\ &= x\sqrt{1-x^2} - (\sqrt{1-x^2})x \\ &= \boxed{0}\end{aligned}$$

d. $\cos(2\sin^{-1} x) = \cos\theta$, where $\theta = \sin^{-1} x$ or $\sin\theta = x$

$\cos(2\theta) = 1 - 2\sin^2\theta = \boxed{1-2x^2}$

5. A $\sin(x-y)\cos y + \cos(x-y)\sin y = \sin\left((x-y)+y\right) = \sin x$