

Repeated Doses of Ritalin

Johnnie Jumper takes 15 mg of Ritalin three times a day. His body metabolizes 25% of it between each dose, so only 75% of the drug is present by the time he takes the next dose. Suppose Q_n is the amount of drug present in his body after the n th dose.

1. Complete the boxes with numbers.

$$Q_1 = \boxed{}$$

$$Q_2 = 0.75Q_1 + 15 = 0.75 \cdot \boxed{} + \boxed{}$$

$$Q_3 = 0.75Q_2 + 15$$

$$= 0.75(0.75 \cdot \boxed{} + \boxed{}) + 15$$

$$= 0.75^2 \cdot \boxed{} + 0.75 \cdot \boxed{} + 15$$

2. Look at the above pattern and write the numerical formula for Q_4 (Your answer involves 0.75, 15 and some + signs)

$$Q_4 =$$

Write the formula for Q_{59} using sigma notation.

$$Q_{59} = \sum_{\boxed{}}^{\boxed{}} \boxed{}$$

3. Since

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

complete the boxes to determine the amount of Ritalin in his body after the 59th dose:

$$\frac{\boxed{}(1 - (\boxed{})^{59})}{\boxed{}} = \boxed{}(1 - (\boxed{})^{59}) \approx \boxed{}$$

4. If $0 < r < 1$ and n gets extremely large (approaches infinity), what happens to
- the value of r^n ? It approaches the value _____
 - the value of $1 - r^n$? It approaches the value _____
 - the value of $\frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \cdot (1-r^n)$? It approaches the value _____
5. You can solve for the stabilization point for Johnnie's Ritalin regimen by noticing that we have $Q_n = 0.75Q_{n-1} + 15$. When stabilization occurs, the next dose is the same as the previous one, so $Q_n = Q_{n-1}$.
- For Johnnie, if stabilization occurs at the value w , then $w = 0.75w + 15$. Solve for w .
 - In the equation $w = 0.75w + 15$, replace 0.75 by the variable r and 15 by the letter a and solve for w for the general situation.

Name _____

Row: ____ Section: 1:30 2:30

A Prescribed Ritalin Regimen

Jimmy Jumper (Johnnie's Dad) also takes Ritalin three times a day.

An adult's body eliminates about $\frac{2}{3}$ of the amount of Ritalin in the system between each dose (and thus keeps $\frac{1}{3}$).

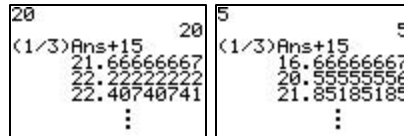
1. Determine what happens to the long-term amount of Ritalin in Jimmy's adult body if he starts with a 9 mg. initial dose and thereafter takes regular doses of 15 mg.

Do this by using the `Ans` key on your calculator repeatedly:

Record the stabilization point.



2. Explore what happens if Jimmie's first dose is 20 mg and all remaining doses are 15 mg. Try different values for a first dose, i.e., 5 mg., 12 mg., 30 mg., 100 mg., etc. (and all remaining doses are 15 mg.)



3. Does the stabilization point depend on the initial dose? Why or why not?
Hint: think about your solution to Question 5 on the back of this sheet.

4. Suppose the Jumper family doctor decides to modify the prescription so that the long-term amount of Ritalin in Jimmy's system will be about 18 mg. How should the prescription be modified? Remember Jimmy eliminates $\frac{2}{3}$ of the Ritalin between every dose. Explain your method.