

## Repeated Doses of Ritalin

Johnnie Jumper takes 15 mg of Ritalin three times a day. His body metabolizes 25% of it between each dose, so only 75% of the drug is present by the time he takes the next dose. Suppose  $Q_n$  is the amount of drug present in his body after the  $n$ th dose.

1. Complete the boxes with numbers.

$$Q_1 = \boxed{\phantom{00}}$$

$$Q_2 = 0.75Q_1 + 15 = 0.75 \cdot \boxed{\phantom{00}} + \boxed{\phantom{00}}$$

$$Q_3 = 0.75Q_2 + 15$$

$$= 0.75(0.75 \cdot \boxed{\phantom{00}} + \boxed{\phantom{00}}) + 15$$

$$= 0.75^2 \cdot \boxed{\phantom{00}} + 0.75 \cdot \boxed{\phantom{00}} + 15$$

2. Look at the above pattern and write the numerical formula for  $Q_4$  (Your answer involves 0.75, 15 and some + signs)

$$Q_4 =$$

Write the formula for  $Q_{59}$  using sigma notation.

$$Q_{59} = \sum_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}} \boxed{\phantom{0000}}$$

3. Since

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

complete the boxes to determine the amount of Ritalin in his body after the 59th dose:

$$\frac{\boxed{\phantom{00}}(1 - (\boxed{\phantom{00}})^{59})}{\boxed{\phantom{00}}} = \boxed{\phantom{00}}(1 - (\boxed{\phantom{00}})^{59}) \approx \boxed{\phantom{000000}}$$

4. If  $0 < r < 1$  and  $n$  gets extremely large (approaches infinity), what happens to
- the value of  $r^n$ ? It approaches the value \_\_\_\_\_
  - the value of  $1 - r^n$ ? It approaches the value \_\_\_\_\_
  - the value of  $\frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \cdot (1-r^n)$ ? It approaches the value \_\_\_\_\_
5. You can solve for the stabilization point for Johnnie's Ritalin regimen by noticing that we have  $Q_n = 0.75Q_{n-1} + 15$ . When stabilization occurs, the next dose is the same as the previous one, so  $Q_n = Q_{n-1}$ .
- For Johnnie, if stabilization occurs at the value  $w$ , then  $w = 0.75w + 15$ . Solve for  $w$ .
  - In the equation  $w = 0.75w + 15$ , replace 0.75 by the variable  $r$  and 15 by the letter  $a$  and solve for  $w$  for the general situation.

Name \_\_\_\_\_

**WR 5: A Prescribed Ritalin Regimen Due: 11-14-08**

Row: \_\_\_\_ Section: 11:00 1:30

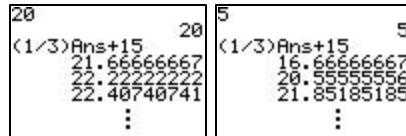
Jimmy Jumper (Johnnie's Dad) also takes Ritalin three times a day. An adult's body eliminates about  $\frac{2}{3}$  of the amount of Ritalin in the system between each dose (and thus keeps  $\frac{1}{3}$ ).

1. Determine what happens to the long-term amount of Ritalin in Jimmy's adult body if he starts with a 9 mg. initial dose and thereafter takes regular doses of 15 mg.

Do this by using the `Ans` key on your calculator repeatedly. Record the stabilization point.



2. Explore what happens if Jimmie's first dose is 20 mg and all remaining doses are 15 mg. Try different values for a first dose, i.e., 5 mg., 12 mg., 30 mg., 100 mg., etc. (and all remaining doses are 15 mg.)



3. Does the stabilization point depend on the initial dose? Why or why not? Hint: think about your solution to Question 5 on the back of this sheet.
4. Suppose the Jumper family doctor decides to modify the prescription so that the long-term amount of Ritalin in Jimmy's system will be about 18 mg. How should the prescription be modified? Remember Jimmy eliminates  $\frac{2}{3}$  of the Ritalin between every dose. Explain your method.