1.a $\cos(-\theta) = \cos(0-\theta)$

$$= \cos(0)\cos\theta + \sin(0)\sin\theta$$
$$= (1)\cos\theta + (0)\sin\theta$$
$$= \cos\theta$$

b. $\sin(\frac{\pi}{2}-\theta) = \sin(\frac{\pi}{2})\cos\theta - \cos(\frac{\pi}{2})\sin\theta$

$$= (1)\cos\theta - (0)\sin\theta$$
$$= \cos\theta$$

c. $\cos(\frac{\pi}{2}-\theta) = \cos(\frac{\pi}{2})\cos\theta + \sin(\frac{\pi}{2})\sin\theta$

$$= (0)\cos\theta + (1)\sin\theta$$
$$= \sin\theta$$

d. $\tan(\pi+\theta) = \dfrac{\tan\pi + \tan\theta}{1-\tan\pi\tan\theta}$

$$= \frac{0+\tan\theta}{1-(0)\tan\theta} = \tan\theta$$

e. $\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$$= 2\sin\theta\cos\theta$$

f. $\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$$= \cos^2\theta - \sin^2\theta$$

g. $\tan(\theta+\theta) = \dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$

$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

h. $\sin(\theta+\pi) = \sin\theta\cos\pi + \cos\theta\sin\pi$

$$= (\sin\theta)(-1) + (\cos\theta)(0)$$
$$= -\sin\theta$$

i. $\cos(\theta-\pi) = \cos\theta\cos\pi + \sin\theta\sin\pi$

$$= (\cos\theta)(-1) + (\sin\theta)(0)$$
$$= -\cos\theta$$

j. $\sin(\frac{\pi}{4}+\theta) + \cos(\frac{\pi}{4}+\theta) = \left[\sin(\frac{\pi}{4})\cos\theta + \cos(\frac{\pi}{4})\sin\theta\right] + \left[\cos(\frac{\pi}{4})\cos\theta - \sin(\frac{\pi}{4})\sin\theta\right]$

$$= \left[\frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta\right] + \left[\frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta\right]$$
$$= \frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\cos\theta$$
$$= \sqrt{2}\cos\theta$$

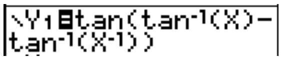
2a. $$\tan(\arctan x \;-\; \arctan x^{-1}) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

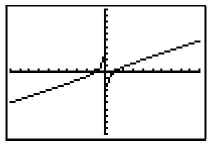
$$= \frac{x-\frac{1}{x}}{1+x\cdot\frac{1}{x}}$$

$$= \frac{x-\frac{1}{x}}{1+1}\cdot\frac{x}{x} = \frac{x^2-1}{2x}$$

Let $\alpha = \arctan x$ and $\beta = \arctan x^{-1}$

Then $\tan\alpha = x$ and $\tan\beta = x^{-1} = \frac{1}{x}$





b. $$\sin(\arcsin x \;-\; \arccos x) = \sin(\alpha-\beta)$$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= (x)(x) - (\sqrt{1-x^2})(\sqrt{1-x^2})$$

$$= x^2-(1-x^2)$$

$$= x^2-1+x^2 = 2x^2-1$$

Let $\alpha = \arcsin x$ and $\beta = \arccos x$

Then $\sin\alpha = x$ and $\cos\beta = x$

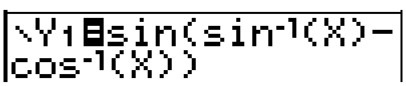
$\cos\alpha = \sqrt{1-x^2}$ and $\sin\beta = \sqrt{1-x^2}$

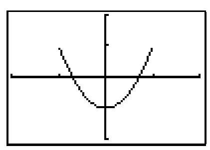
Note: $\cos\alpha = \sqrt{1-\sin^2\alpha}$

$= \sqrt{1-x^2}$

$\sin\beta = \sqrt{1-\cos^2\beta}$

$= \sqrt{1-x^2}$





c. $$\cos(\arcsin x \;+\; \arccos x) = \cos(\alpha+\beta)$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

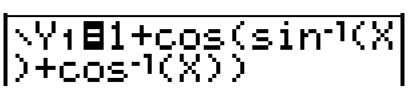
$$= (\sqrt{1-x^2})(x) - (x)(\sqrt{1-x^2})$$

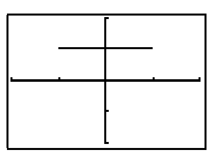
$$= 0$$

Let $\alpha = \arcsin x$ and $\beta = \arccos x$

Then $\sin\alpha = x$ and $\cos\beta = x$

$\cos\alpha = \sqrt{1-x^2}$ and $\sin\beta = \sqrt{1-x^2}$





d. $\cos(\arcsin x - \arccos x) = \cos(\alpha - \beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

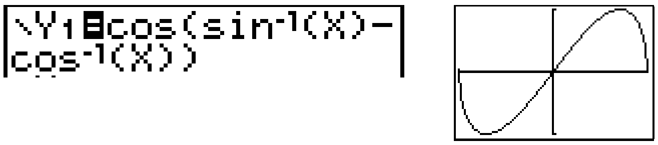
$= (\sqrt{1-x^2})(x) + (x)(\sqrt{1-x^2})$

$= 2x\sqrt{1-x^2}$

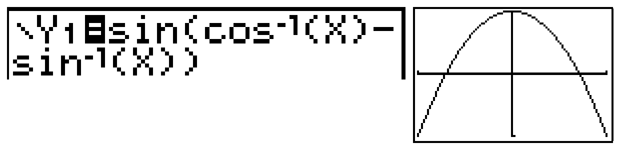
Let $\alpha = \arcsin x$ and $\beta = \arccos x$

Then $\sin\alpha = x$ and $\cos\beta = x$

$\cos\alpha = \sqrt{1-x^2}$ and $\sin\beta = \sqrt{1-x^2}$



e. $(\arccos x - \arcsin x)$ and
$(\arcsin x - \arccos x)$ are opposites.
Since $\sin(\theta) = -\sin(\theta)$,
$\sin(\alpha - \beta) = -\sin(\beta - \alpha)$

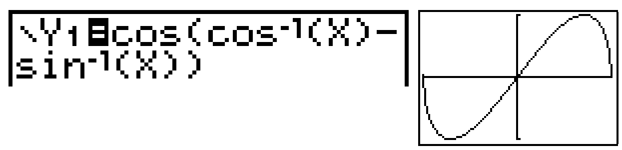


$\sin(\arccos x - \arcsin x) = 1 - 2x^2$

The graphs of (b) and (e) are vertical reflections of each other since they generate opposite outputs.

f. Since $\cos(-\theta) = \cos(\theta)$, $\cos(\alpha - \beta) = \cos(\beta - \alpha)$

$\cos(\arccos x - \arcsin x) = 2x\sqrt{1-x^2}$



g. Graphing this expression would result in an error since it is undefined for all real numbers.

$$\tan(\arctan x + \arctan x^{-1}) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{x + \frac{1}{x}}{1 - x\cdot\frac{1}{x}}$$

$$= \frac{x + \frac{1}{x}}{1-1}$$

$$= \frac{x + \frac{1}{x}}{0} \text{ which is undefined.}$$

Let $\alpha = \arctan x$ and $\beta = \arctan x^{-1}$

Then $\tan\alpha = x$ and $\tan\beta = x^{-1} = \frac{1}{x}$