

Using Sum and Difference Identities

1. Use the sum and difference identities to show the following facts, some old, some new.

Example: $\sin(-\theta) = -\sin\theta$

Answer: $\sin(-\theta) = \sin(0 - \theta) = \sin 0 \cos\theta - \cos 0 \sin\theta = 0 \cdot \cos\theta - 1 \cdot \sin\theta = -\sin\theta$

a. $\cos(-\theta) = \cos\theta$

b. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

c. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

d. $\tan(\pi + \theta) = \tan\theta$

e. $\sin 2\theta = 2 \sin\theta \cos\theta$
Hint: $\sin 2\theta = \sin(\theta + \theta)$

f. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

g. $\tan 2\theta = \frac{2 \tan\theta}{1 - \tan^2\theta}$

h. $\sin(\theta + \pi) = -\sin\theta$

i. $\cos(\theta - \pi) = -\cos\theta$

j. $\sin\left(\frac{\pi}{4} + \theta\right) + \cos\left(\frac{\pi}{4} + \theta\right) = \sqrt{2} \cos\theta$