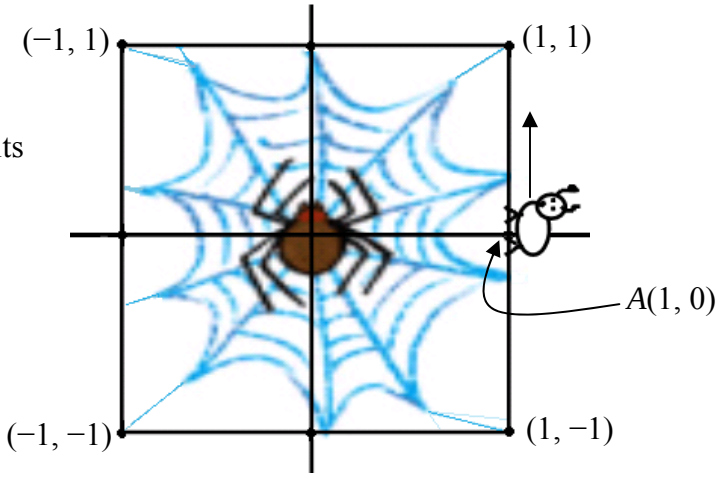
Writing Assignment 2: Bug on a Square Track
Due: Wednesday, September 10, 2008 (45 points)

Name _____
Section: 11:00 1:30

Suppose a bug travels ***counterclockwise*** around the square track shown:

Assume the bug begins at $A(1, 0)$ and walks t units along the square, carefully staying on the track. A large hairy spider is fixed at the origin, ready to eat him if he gets off the track.

As long as the bug remains on the track, the spider will not touch him!
(Geneva Convention).



For example: If the bug walks $t = 0.5$ units from A counterclockwise, the bug will be at the point (1, 0.5).

(1) 1. a. If the bug walks $t = 1.5$ units from A counterclockwise, where will its location be? (____, ____)

(4) b. List the ordered pairs for which the bug is **FARTHEST** from the spider? There are 4 ordered pairs.

(____, ____) , (____, ____) , (____, ____), and (____, ____)

(4) c. List the ordered pairs for which the bug is **CLOSEST** to the spider? There are 4 ordered pairs.

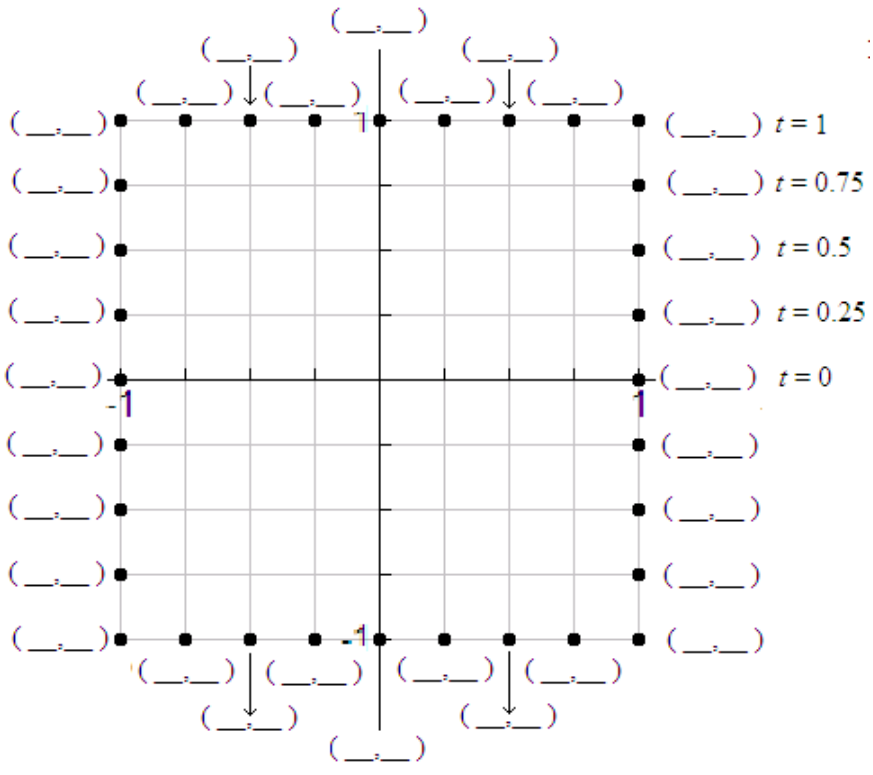
(____, ____) , (____, ____) , (____, ____), and (____, ____)

(4) 2. The function $S(t)$ gives the y-coordinate of the location of the bug on the square after it has traveled t units from $A(1,0)$ counterclockwise. Complete the table for $S(t)$.

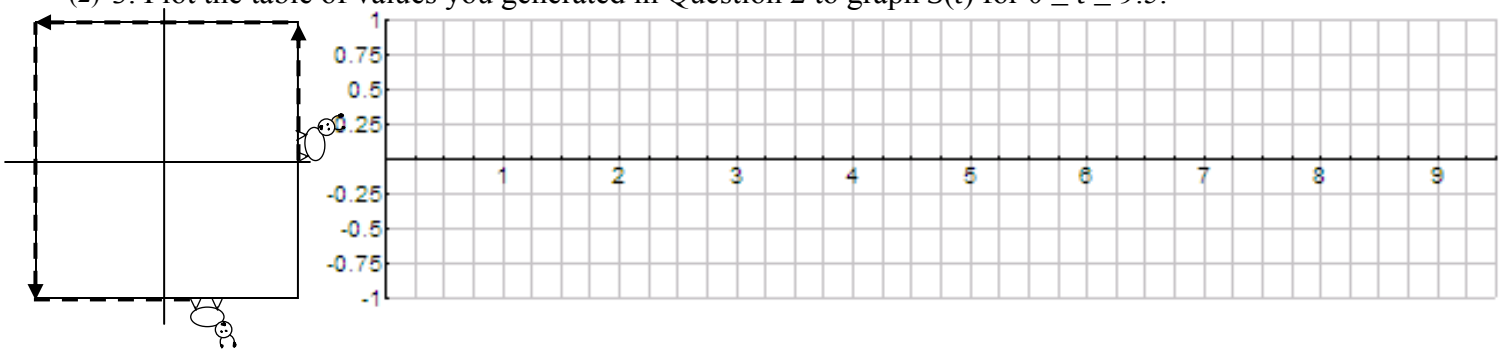
t	$S(t)$
0	
0.25	
0.5	
0.75	
1	
1.25	
1.5	
1.75	
2	
2.25	
2.5	
2.75	
3	
3.25	
3.5	
3.75	
4	
4.25	
4.5	
4.75	

t	$S(t)$
5	
5.25	
5.5	
5.75	
6	
6.25	
6.5	
6.75	
7	
7.25	
7.5	
7.75	
8	
8.25	
8.5	
8.75	
9	
9.25	
9.5	

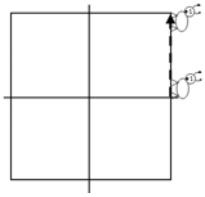
Hint: It may be helpful to write the ordered pairs on the diagram below, and include the corresponding values of t.



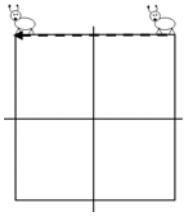
(2) 3. Plot the table of values you generated in Question 2 to graph $S(t)$ for $0 \le t \le 9.5$.



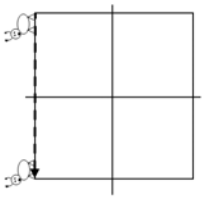
(5) 4. As the bug journeys one circuit counterclockwise around the square, determine whether the height $S(t)$ of the bug above the x-axis is increasing, decreasing, or is constant and report the interval of t for each part below by completing the blanks. Observe how your answers correspond to your graph in the previous question.

a. 

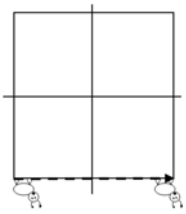
On the interval from $t = 0$ to $t =$ ___, $S(t)$ is _____
(increasing, decreasing, constant)

b. 

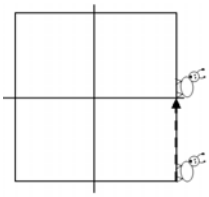
On the interval from $t =$ ___ to $t =$ ___, $S(t)$ is _____
(increasing, decreasing, constant)

c. 

On the interval from $t =$ ___ to $t =$ ___, $S(t)$ is _____
(increasing, decreasing, constant)

d. 

On the interval from $t =$ ___ to $t =$ ___, $S(t)$ is _____
(increasing, decreasing, constant)

e. 

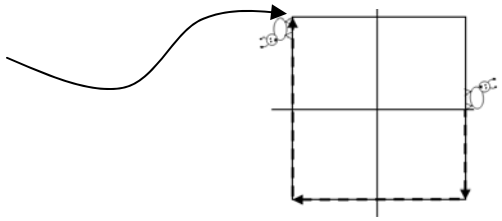
On the interval from $t =$ ___ to $t = 8$, $S(t)$ is _____
(increasing, decreasing, constant)

(2) 5. To make $S(t)$ defined for ***all*** real numbers, we will interpret the outputs of $S(t)$ as follows:

- For values of $t \ge 0$, the bug moves *forward* (in a ***counterclockwise*** direction) around the square a total of t units. $S(t)$ is its y-coordinate at this location.
- For values of $t < 0$, the bug moves *backward* (in a ***clockwise*** direction) around the square a total of t units. (It's a talented bug.) $S(t)$ is its y-coordinate at this location.

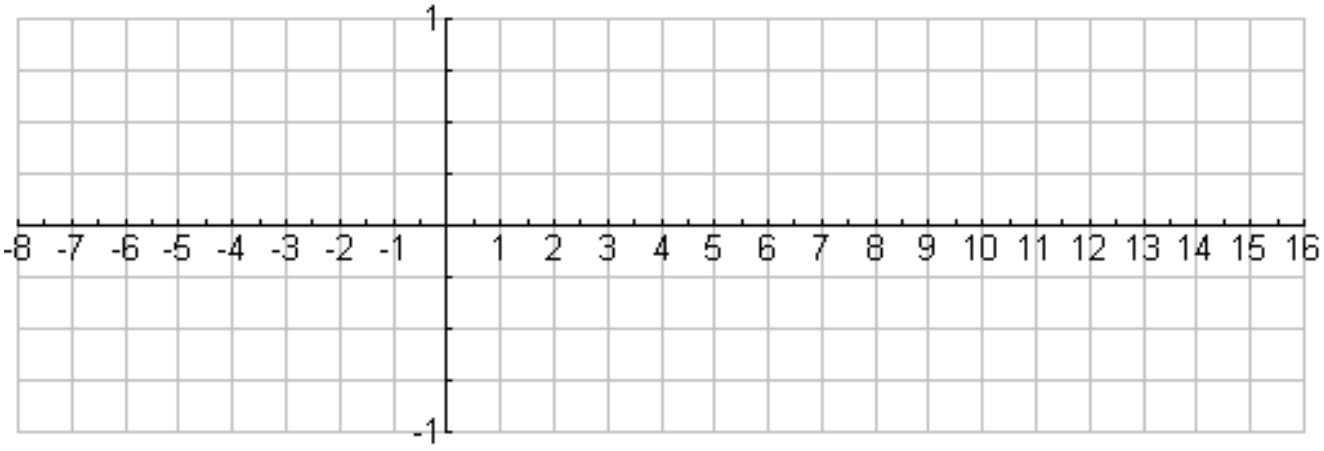
For example, suppose that the bug walks *backward* in a ***clockwise*** direction around the square a total of 5 units, as shown in the figure.
Since we are walking backwards, we indicate this as $t = -5$. Thus the value of $S(-5) = 1$.

There are many values of t which put the bug at the same location in that upper left corner of the square.
Complete the blanks with two values of t that are greater than 0.
$S(-5) = S($ _____ $) = S($ _____ $) = 1$



6. $S(t)$ is now defined for **all** values of t, both positive, negative, and 0.

(1) a. On the axes below, graph $S(t)$ for $-8 \le t \le 16$. Hint: You have already sketched it for $0 \le t \le 9.5$, so just recopy that part on the axes below and extend the graph, keeping in mind your answers to the previous question.



(2) b. Mark points on the above graph corresponding to $S(-5)$ and the two values of t you reported in Question 5.

7. The domain of $S(t)$ is all real numbers.

(2) a. What is the range? Complete the boxes: $\square \le S(t) \le \square$

(1) b. When the bug completes one cycle, the graph of $S(t)$ repeats. The length of a cycle is called the period, and $S(t)$ is called a periodic graph. What is the period of $S(t)$? _____

(1) c. In one period of $S(t)$, the graph changes its pattern when the bug turns a corner on the square. In the **first period** of $S(t)$, what are these four values of t? $t =$ ___ , ___ , ___ , and ___ .

(1) d. When the height of the bug changes from *increasing to constant* or *constant to decreasing* or *decreasing to constant* or *constant to increasing*, (i.e., when the bug turns a corner on the square), is the graph of $S(t)$ **smooth** at these four values of t or **sharp**? _____

(1) e. Try to explain your choice for part d.

(2) f. Find the values of each:
$S(28) =$ _____ $S(-305) =$ _____

(1) g. Is $S(t)$ an ***odd*** function, ***even***, or ***neither even nor odd***? _____ (See Section 5.2)

(2) 8. The function $C(t)$ gives the x-coordinate of the location of the bug on the square after it has traveled t units from $A(1,0)$ counterclockwise. Complete the table for $C(t)$. With the table to guide you, plot the table of values you generated to graph $C(t)$ for $-8 \le t \le 16$.

t	$C(t)$
0	
0.25	
0.5	
0.75	
1	

t	$C(t)$
1.25	
1.5	
1.75	
2	
2.25	

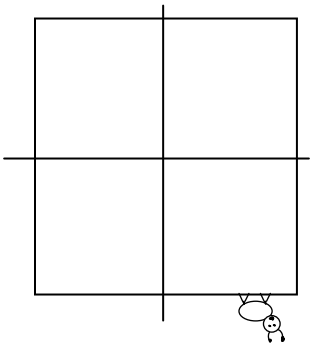
t	$C(t)$
2.5	
2.75	
3	
3.25	
3.5	

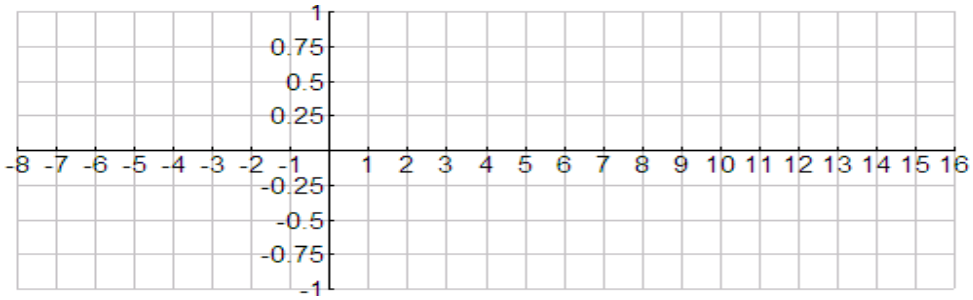
t	$C(t)$
3.75	
4	
4.25	
4.5	
4.75	

t	$C(t)$
5	
5.25	
5.5	
5.75	
6	

t	$C(t)$
6.25	
6.5	
6.75	
7	
7.25	

t	$C(t)$
7.5	
7.75	
8	





9. The domain of $C(t)$ is all real numbers.

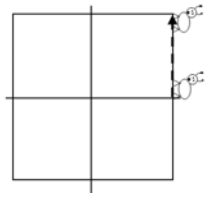
(2) a. What is the range? Complete the boxes: $\square \le C(t) \le \square$ _____

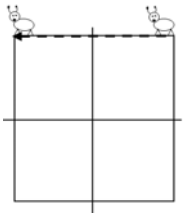
(1) b. What is the period? _____

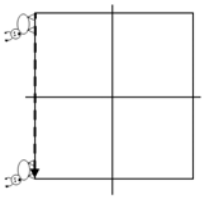
(2) c. Find the values of each:
$C(151)$ = _____ $C(-33.25)$ = _____

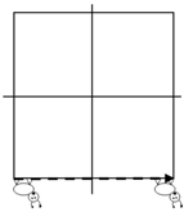
(1) d. Is $C(t)$ an ***odd*** function, ***even***, or ***neither even nor odd***? _____

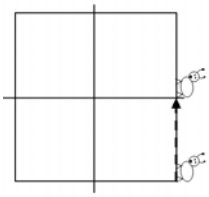
(5) 10. As the bug journeys one circuit counterclockwise around the square, determine whether $C(t)$ is increasing, decreasing, or is constant and report the interval of t for each part below by completing the blanks. Observe how your answers correspond to your graph in the previous question.

a. 
On the interval from $t = 0$ to $t =$ ___, $C(t)$ is _____
(increasing, decreasing, constant)

b. 
On the interval from $t =$ ___ to $t =$ ___, $C(t)$ is _____
(increasing, decreasing, constant)

c. 
On the interval from $t =$ ___ to $t =$ ___, $C(t)$ is _____
(increasing, decreasing, constant)

d. 
On the interval from $t =$ ___ to $t =$ ___, $C(t)$ is _____
(increasing, decreasing, constant)

e. 
On the interval from $t =$ ___ to $t = 8$, $C(t)$ is _____
(increasing, decreasing, constant)