

### In-Class Activity: Repeated Doses of Ritalin

Johnnie Jumper takes 15 mg of Ritalin three times a day. His body metabolizes 25% of it between each dose, so only 75% of the drug is present by the time he takes the next dose. Suppose  $Q_n$  is the amount of drug present in his body after the  $n$ th dose.

1. Complete the boxes with numbers.

$$Q_1 = \boxed{\phantom{00}}$$

$$Q_2 = 0.75Q_1 + 15 = 0.75 \cdot \boxed{\phantom{00}} + \boxed{\phantom{00}}$$

$$Q_3 = 0.75Q_2 + 15$$

$$= 0.75(0.75 \cdot \boxed{\phantom{00}} + \boxed{\phantom{00}}) + 15$$

$$= 0.75^2 \cdot \boxed{\phantom{00}} + 0.75 \cdot \boxed{\phantom{00}} + 15$$

2. Look at the above pattern and write the numerical formula for  $Q_4$  (Your answer involves 0.75, 15 and some + signs)

$$Q_4 =$$

Write the formula for  $Q_{59}$  using sigma notation.

$$Q_{59} = \sum_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}} \boxed{\phantom{00}}$$

3. Since

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

complete the boxes to determine the amount of Ritalin in his body after the 59th dose:

$$\frac{\boxed{\phantom{00}}(1 - (\boxed{\phantom{00}})^{59})}{\boxed{\phantom{00}}} = \boxed{\phantom{00}}(1 - (\boxed{\phantom{00}})^{59}) \approx \boxed{\phantom{00}}$$

4. If  $0 < r < 1$  and  $n$  gets extremely large (approaches infinity), what happens to
- the value of  $r^n$ ? It approaches the value \_\_\_\_\_
  - the value of  $1 - r^n$ ? It approaches the value \_\_\_\_\_
  - the value of  $\frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \cdot (1-r^n)$ ? It approaches the value \_\_\_\_\_
5. You can solve for the stabilization point for Johnnie's Ritalin regimen by noticing that we have  $Q_n = 0.75Q_{n-1} + 15$ . When stabilization occurs, the next dose is the same as the previous one, so  $Q_n = Q_{n-1}$ .
- For Johnnie, if stabilization occurs at the value  $w$ , then  $w = 0.75w + 15$ . Solve for  $w$ .
  - In the equation  $w = 0.75w + 15$ , replace 0.75 by the variable  $r$  and 15 by the letter  $a$  and solve for  $w$  for the general situation.

Name \_\_\_\_\_

Row: \_\_\_\_ Section: 9:00 11:00

**WR 6: A Prescribed Ritalin Regimen Due 11-13**

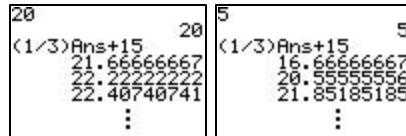
Jimmy Jumper (Johnnie's Dad) also takes Ritalin three times a day. An adult's body eliminates about  $\frac{2}{3}$  of the amount of Ritalin in the system between each dose (and thus keeps  $\frac{1}{3}$ ).

1. (2 pts) Determine what happens to the long-term amount of Ritalin in Jimmy's adult body if he starts with a 9 mg. initial dose and thereafter takes regular doses of 15 mg.

Do this by using the `Ans` key on your calculator repeatedly. Record the stabilization point.



2. (2 pts) What happens if Jimmie's first dose is 20 mg and all remaining doses are 15 mg. ? Try different values for a first dose, i.e., 5 mg., 12 mg., 30 mg., 100 mg., etc. (and all remaining doses are 15 mg.) What happens?



3. (2 pts) Does the stabilization point depend on the initial dose? Why or why not? Hint: think about your solution to Question 5 on the back of this sheet.

4. (4 pts) Suppose the Jumper family doctor decides to modify the prescription so that the long-term amount of Ritalin in Jimmy's system will be about 18 mg. How should the prescription be modified? Remember Jimmy eliminates  $\frac{2}{3}$  of the Ritalin between every dose. Explain your method.