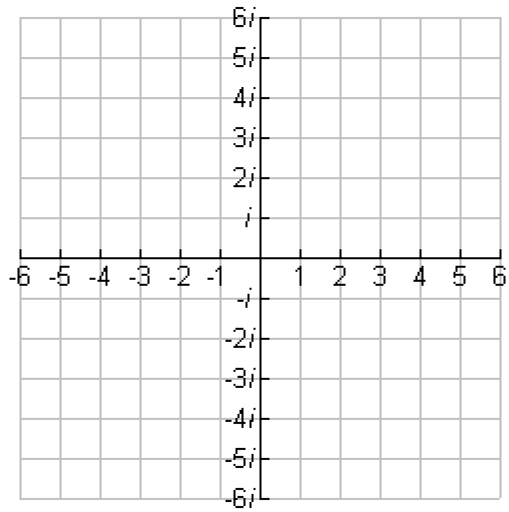
Review for the MA 154 Final Fall 2008 Date: _______________Time:_______________ Room:________



1. Write each of the complex numbers z_1, z_2, and z_3 in the form $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$.
Write z_4 and z_5 in rectangular form $z = x + yi$. Exact please.
Plot and label each number on the complex plane.

(a) $z_1 = 4$ **(b)** $z_2 = 5 - 5i$ **(c)** $z_3 = 3i$

(d) $z_4 = 3e^{-i\pi} = 3\cos(-\pi) + i3\sin(-\pi)$

(e) $z_5 = \sqrt{2}e^{i3\pi/4} = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$

2. Perform the complex number arithmetic and write your answer in rectangular form $x + yi$.

(a) $2(4 + 2i) - (3 - i)$ **(b)** $(2i)(3 - i)$ **(c)** $\dfrac{i+1}{i}$

(d) $(2e^{-i5\pi/2})^{10}$ **(e)** $(1 + i)^{14}$

3. Eliminate the parameter and write a formula which only involves x and y. Give the domain and range.

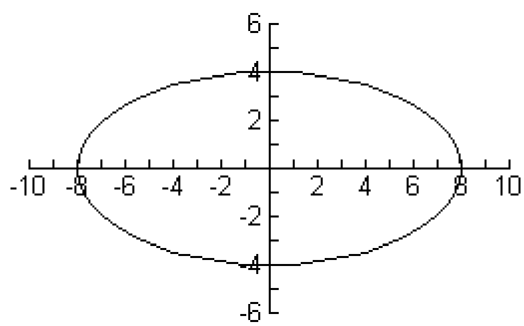
(a) $x = 2\cos t,\ y = -3\cos t + 3$ **(b)** $x = 4 + \cos t,\ y = 4\cos^2 t$ **(c)** $x = \cos 2t,\ y = 2\cos t$
(Hint: consider a double angle identity)

4. Eliminate the parameter and write a formula which only involves x and y.

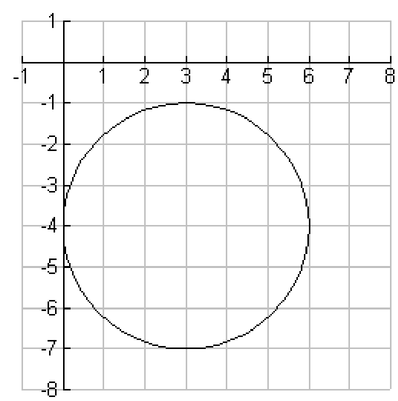
(a) $x = t - 1,\ y = e^t$ **(b)** $x = e^{0.5t},\ y = e^t$

5. Write implicit formulas for each of the conics graphed below. Give the center, focal points (exact and approximate to two decimal places) and vertices if any. For parts (a) through (c), give the parametric formulas as well. For part (d), report the equations of the asymptotes.

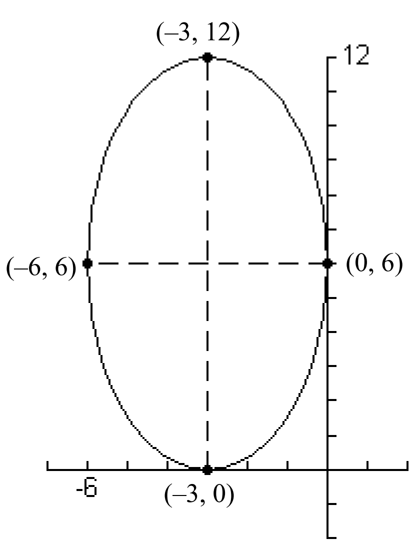
(a)



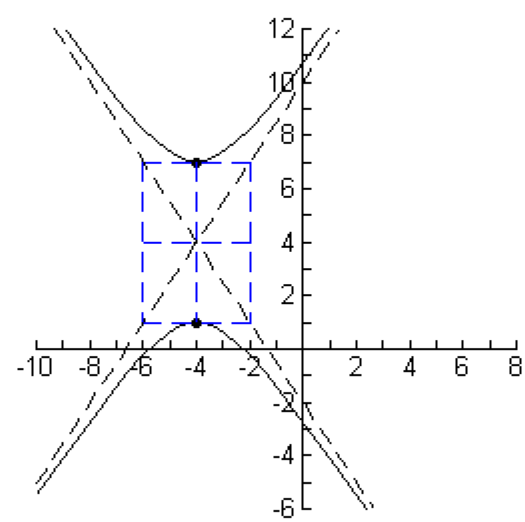
(b)



(c)



(d)



6. Sketch a graph of each of the conics. Report the vertices (or vertex) and focal point(s) and plot them. For parts **(a)** - **(d)** give the center. For parts **(e)** and **(f)** give the equation of the directrix.

a. $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$

b. $\dfrac{(x-1)^2}{25} + \dfrac{(y+4)^2}{9} = 1$

c. $\dfrac{y^2}{25} - \dfrac{x^2}{100} = 1$

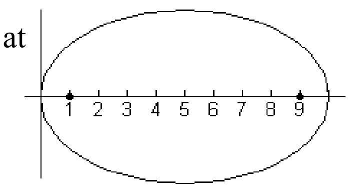
d. $\dfrac{(x+2)^2}{1} - \dfrac{(y-5)^2}{4} = 1$

e. $y^2 = 8x$

f. $x^2 = -\frac{1}{4}(y+1)$

7. A parabola has its vertex at (0, 0) and its focus at (0, 3). Give its equation.

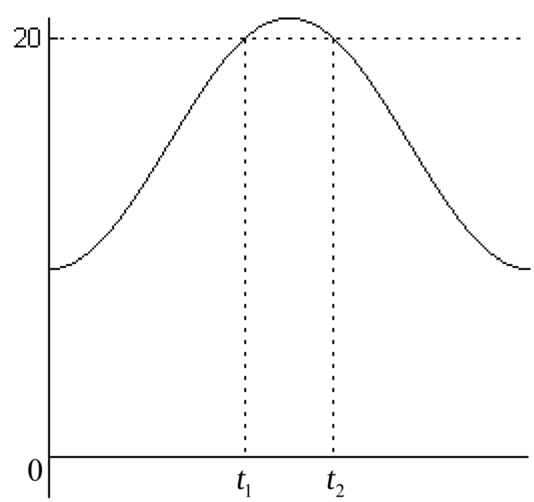
8. The ellipse shown has focal points at (1, 0) and (9, 0). Give its equation.



9. **For Question 9, a day is a 24 hour period beginning at midnight.**
Two species of plant, A and B, propagate by dispersing their seeds in the wind during the height of summer. On a typical day in the height of summer, the wind speed, $w(t)$, measured in miles per hour t hours after midnight, is given by the formula $w(t) = -6\cos(\frac{\pi t}{12}) + 15$.

(a) Species A favors propagation in high winds and will only release its seeds if the wind speed is no less than twenty miles per hour.
In the figure, the graph of $w(t)$ is given and the time interval over which species A will release its seeds is also indicated.

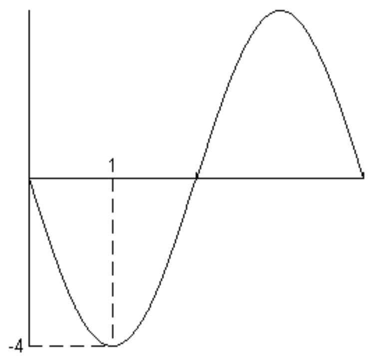
Complete the figure by calculating the values of the two endpoints of the interval.



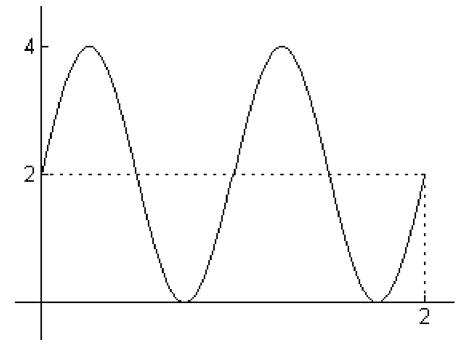
(b) The seeds of species B are destroyed in high winds and hence species B has a seed release mechanism which only releases seeds if the wind speed is no greater than ten miles per hour. For how many hours, on a typical day in high summer, will species B be releasing seeds?

10. Find a possible formula for each:

a.



b.



Additional Problems from Text

Section 6.6: 17, 19, 23
Section 6.7: 37, 43, 51
Ch 6 Tools: 1, 3, 23
Chapter 6 Review: 3, 15, 43-47 odd, 61,

Section 7.1: 5, 11, 17, 39
Section 7.2: 3, 5, 17, 41, 45
Section 7.6: 1-14, 17-25
Chapter 7 Review: 3, 5, 7, 9

Section 10.1: 5, 12
Section 10.2: 3, 13, 14, 22
Section 10.3: 12, 15
Section 10.4: 1, 25
Chapter 10 Review: 5, 11, 29, 35

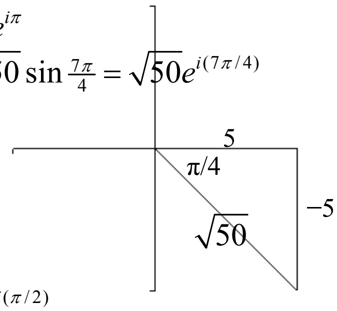
Section 11.1: 7-25 odd, 30-33
Section 11.2: 3, 13, 19, 23, 35, 41
Section 11.3: 1-15 odd
Section 11.4: 1-9 odd, 19
Chapter 11 Review: 1, 3, 5, 7, 9, 10

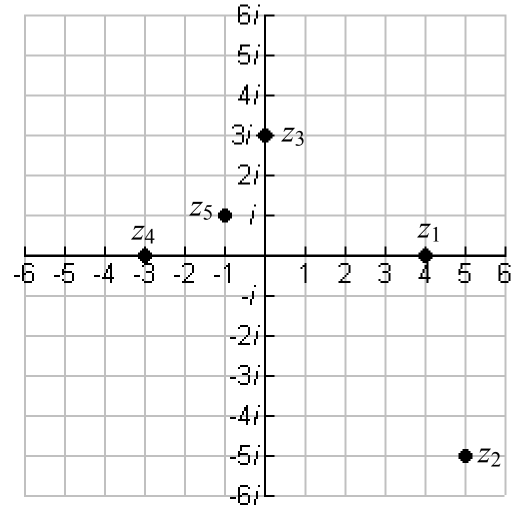
Section 12.1: 1-11 odd, 28
Section 12.2: 1, 13a, 15, 17
Section 12.3: 1, 3, 17a
Section 12.4: 1, 3, 11a
Section 12.5: 1-23 odd, 25, 26, 27, 30, 31
Chapter 12 Review: 5, 9, 17-19, 28

Answers to Review

1. a. $z_1 = 4 = 4\cos\pi + i4\sin\pi = 4e^{i\pi}$

   b. $z_2 = 5 - 5i = \sqrt{50}\cos\frac{7\pi}{4} + i\sqrt{50}\sin\frac{7\pi}{4} = \sqrt{50}e^{i(7\pi/4)}$





   c. $z_3 = 3i = 3\cos\frac{\pi}{2} + i3\sin\frac{\pi}{2} = 3e^{i(\pi/2)}$

   d. $z_4 = 3e^{-i\pi} = 3\cos(-\pi) + i3\sin(-\pi) = 3(-1) + 3(0) = -3.$

   e. $z_5 = \sqrt{2}e^{i3\pi/4} = \sqrt{2}(-\frac{1}{\sqrt{2}}) + i\sqrt{2}(\frac{1}{\sqrt{2}}) = -1 + i$

2. a. $2(4 + 2i) - (3 - i) = 8 + 4i - 3 + i = 5 + 5i$

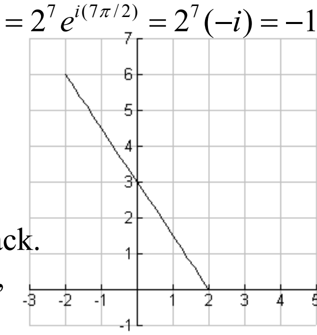
   b. $(2i)(3 - i) = 6i - 2i^2 = 6i - 2(-1) = 6i + 2 = 2 + 6i$

   c. $\dfrac{i+1}{i} = \dfrac{i+1}{i}\cdot\dfrac{i}{i} = \dfrac{i(i+1)}{i^2} = \dfrac{i^2+i}{-1} = \dfrac{(-1)+i}{-1} = 1 - i$

   d. $(2e^{-i5\pi/2})^{10} = 2^{10}e^{-i(5\pi/2)10} = 2^{10}e^{-i(25\pi)} = 2^{10}(\cos(-25\pi) + i\sin(-25\pi)) = 2^{10}(-1) = -1024$

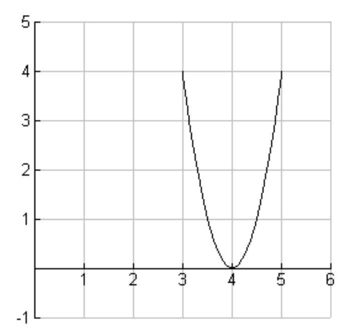
   e. To find $(1 + i)^{14}$ write $1 + i$ in the form $re^{i\theta}$. The angle $\theta = \pi/4$ and distance $r = \sqrt{2}$, so $1 + i = \sqrt{2}e^{i(\pi/4)}$. Then $(\sqrt{2}e^{i(\pi/4)})^{14} = (\sqrt{2})^{14}\cdot e^{i(\pi/4)\cdot 14} = (2^{1/2})^{14}e^{i(7\pi/2)} = 2^7 e^{i(7\pi/2)} = 2^7(-i) = -128i$.

3. a. $y = -\frac{3}{2}x + 3$



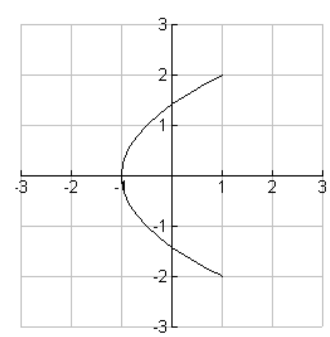
   Since $x = 2\cos t,\ y = -3\cos t + 3$, solve for cos t.
   Therefore $\cos t = \frac{x}{2}$, so $y = -3\cos t + 3 = -3(\frac{x}{2}) + 3 = \frac{-3x}{2} + 3$.
   Using a grapher, we can see that when $t = 0$ the curve starts at the endpoint (2, 0) travels on a line to the endpoint (–2, 6), and repeats back. Note that the domain is restricted to $-2 \le x \le 2$ since $-2 \le 2\cos t \le 2$, and the range is restricted to $0 \le y \le 6$ since $0 \le -3\cos t + 3 \le 6$

   b. $y = 4(x-4)^2$



   Solve for cos t to get $\cos t = x - 4$ and substitute:
   therefore, $y = 4(\cos t)^2 = 4(x-4)^2$ which is a parabola.
   Using a grapher, we can see that when $t = 0$ the curve starts at the endpoint (5, 4), travels on the parabola to the endpoint (3, 4) and repeats back. The domain is $-3 \le x \le 5$ and the range is $0 \le y \le 4$.

   c. $x = \frac{1}{2}y^2 - 1$

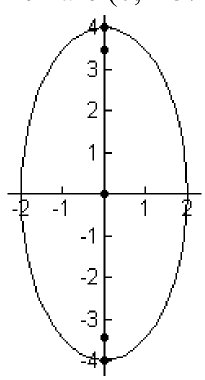


   We must write cos $2t$ in terms of cos t, which can be done if we use the double angle identity $\cos 2t = 2\cos^2 t - 1$. Solving $y = 2\cos t$ for cos t, we have $\cos t = \frac{y}{2}$, and substituting gives $x = 2(\frac{y}{2})^2 - 1$.
   Simplifying gives $x = 2(\frac{y^2}{4}) - 1 = \frac{y^2}{2} - 1$.
   The domain is $-1 \le x \le 1$ and the range is $-2 \le y \le 2$.

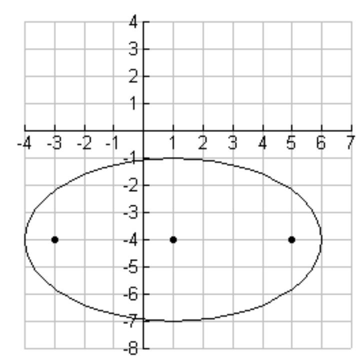
4. Eliminate the parameter and write a formula which only involves x and y.
   (a) $y = e^{x+1}$ since $t = x+1$ and $y = e^t = e^{x+1}$.
   (b) $x = \sqrt{y}$ or $y = x^2, x > 0$ since $x = e^{0.5t}$, $y = e^t$ and $x = (e^t)^{0.5} = y^{0.5} = \sqrt{y}$

5. a. $\frac{x^2}{64} + \frac{y^2}{16} = 1$ center: (0, 0) vertices: ($\pm$ 8, 0) The focal points are c units from the center where $c^2 = 64 - 16 = 48$, so $c = \sqrt{48} \approx 6.93$. So the focal points are ($\pm\sqrt{48}$, 0) or ($\pm$ 6.93, 0).
   Parametric: $x = 8\cos t$, $y = 4\sin t$.
   b. $(x-3)^2 + (y+4)^2 = 9$ center: (3, –4) There are no vertices or focal points (this is a circle).
   Parametric: $x = 3\cos t + 3$, $y = 3\sin t - 4$.
   c. $\frac{(x+3)^2}{9} + \frac{(y-6)^2}{36} = 1$ center: (–3, 6) vertices: (–3, 12) and (–3, 0) We have $c^2 = 36 - 9 = 27$ so $c = \sqrt{27} \approx 5.20$ and the focal points are on the major axis at (–3, $6\pm\sqrt{27}$) or, approximately, at (–3, 0.80) and (–3, 11.20) Parametric: $x = 3\cos t - 3$, $y = 6\sin t - 6$.
   d. $\frac{(y-4)^2}{9} - \frac{(x+4)^2}{4} = 1$ center: (–4, 4) vertices: (–4, 1) and (–4,7) The focal points are c units from the center where $c^2 = 9 + 4 = 13$, so $c = \sqrt{13} \approx 3.61$. So the focal points are $(-4,\ 4\pm\sqrt{13})$ or $(-4,\ 0.39)$ or $(-4,\ 7.61)$. Asymptotes: $(y-4) = \pm\frac{3}{2}(x+4)$ or $y = 1.5x+10$, $y = -1.5x - 2$.

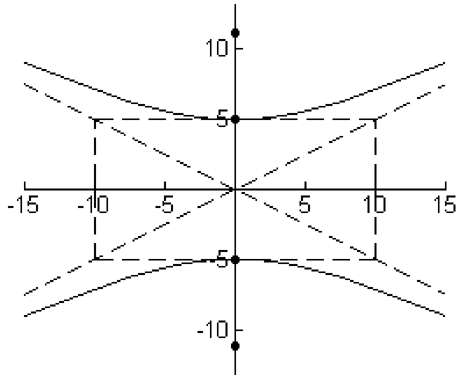
6. a. center: (0, 0)
   vertices: (0, 4), (0, –4)
   focal points: $(0, \pm\sqrt{12})$
   which are (0, ± 3.46)



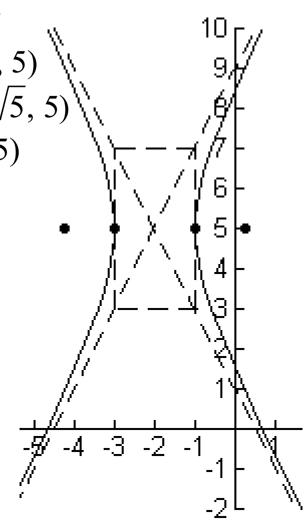
   b. center: (1, –4)
   vertices: (6, –4), (–4, –4)
   focal points: (–3 –4) and (5, –4)
   We have $c = \sqrt{25-9} = 4$ so at $(1\pm 4, -4)$



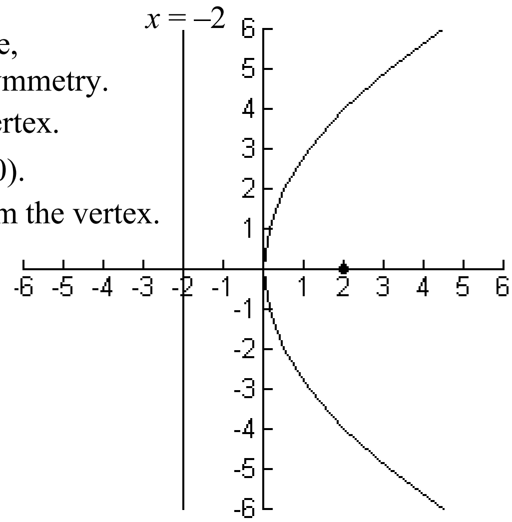
   c. center: (0, 0)
   asymptotes: $y = \pm\frac{5}{10}x$ or $y = \pm\frac{1}{2}x$
   vertices: (0, 5), (0, –5)
   focal points: $(0, \pm\sqrt{125})$ or (0, ± 11.18)



   d. center: (–2, 5)
   asymptotes: $y = \pm 2x$
   vertices: (–1, 5), (–4, 5)
   focal points: $(-2\pm\sqrt{5},\ 5)$
   or (–4.24, 5), (0.24, 5)

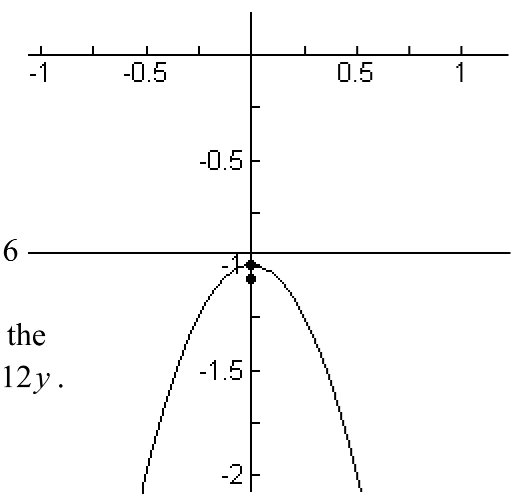


6. e. The vertex is at the origin. Since y is of second degree, we expect the parabola to have a horizontal axis of symmetry. The equation $y^2 = 4px$ has a focus p units from the vertex. The graph of $y^2 = 8x$ has $p = 2$, so the focus is at (2, 0). The directrix is the vertical line $x = -2$, which is 2 from the vertex.



f. The vertex is at (0, –1). Since x is of second degree, we expect the parabola to have a vertical axis of symmetry (similar to $y = x^2$). Here $4p = -\frac{1}{4}$ so $p = -\frac{1}{16}$. The focus is $p = -\frac{1}{16}$ units from the vertex on the vertical axis, or at $(0, -1\frac{1}{16})$ or $(0, -\frac{17}{16})$. The directrix is $p = \frac{1}{16}$ units from the vertex so it has the equation $y = -\frac{15}{16}$.

$y = -15/16$



7. The focus (0, 3) is on the axis of symmetry (vertical), so the parabola has the equation $x^2 = 4py$ with $p = 3$, so $x^2 = 12y$.

8. center: (5, 0) RUN = 5 $c = 4$

Since $c^2 = (\text{RUN})^2 - (\text{RISE})^2 = (5)^2 - (\text{RISE})^2 = 4^2$,

we can solve for the RISE: $4^2 = (5)^2 - (\text{RISE})^2$

$(\text{RISE})^2 = (5)^2 - (4)^2 = 25 - 16 = 9$ Therefore RISE = 3.

$$\frac{(x-5)^2}{25} + \frac{(y)^2}{9} = 1$$

9. a. $t_1 = 9.763,\ t_2 = 14.237$ (Solve graphically)

b. 19.526 hours. Find where $w(t) = 10$. This is $t_1 = 2.237$ and $t_2 = 21.763$, so $t_2 - t_1 \approx 19.526$.

10. a. $y = -4\sin\left(\frac{\pi x}{2}\right)$ b. $y = 2 + 2\sin(2\pi x)$