

1.  $\frac{\cos x}{1 - \sin x} - \tan x =$

- A.
- $\sin x$
- B.
- $\cos x$
- C.
- $\csc x$
- D.
- $\sec x$
- E. None of these.

2.  $\frac{\frac{1}{\cos x} - 1}{1 - \cos x} =$

- A.
- $\sin x$
- B.
- $\cos x$
- C.
- $\csc x$
- D.
- $\sec x$
- E. None of these.

3. Solve on the requested interval.

- a.  $2\cos^2\theta = 3\sin\theta + 3$     on  $[0, 2\pi)$   
b.  $2\cos\theta \sin\theta = \sin\theta$     on  $[0, 2\pi)$   
c.  $2\cos^2\theta - 3\cos\theta = 2$     on  $(-\infty, \infty)$   
d.  $\sin 2\theta = \sin\theta$     on  $[0, 2\pi)$   
e.  $\cos 2\theta = -\cos\theta$     on  $[0, 2\pi)$

4. Write as algebraic expressions

- a.  $\cos(\sin^{-1} 2x)$   
b.  $\cot(\tan^{-1} 2x)$   
c.  $\cos(\cos^{-1} x + \sin^{-1} x)$   
d.  $\cos(2\sin^{-1} x)$

5.  $\sin(x - y)\cos y + \cos(x - y)\sin y =$

- A.
- $\sin x$
- B.
- $\cos x$
- C.
- $\sin y$
- D.
- $\cos y$
- E. None of these.

## ANSWERS

1. D

$$\begin{aligned} \frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} &= \frac{\cos x}{1-\sin x} \cdot \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{1-\sin x}{1-\sin x} = \frac{\cos^2 x - \sin x(1-\sin x)}{(1-\sin x)\cos x} \\ &= \frac{\cos^2 x - \sin x + \sin^2 x}{(1-\sin x)\cos x} = \frac{\cos^2 x + \sin^2 x - \sin x}{(1-\sin x)\cos x} = \frac{(1-\sin x)}{(1-\sin x)\cos x} = \frac{1}{\cos x} = \sec x \end{aligned}$$

2. D

$$\frac{\frac{1}{\cos x} - 1}{1 - \cos x} \cdot \frac{\cos x}{\cos x} = \frac{(\frac{1}{\cos x} - 1)\cos x}{(1 - \cos x)\cos x} = \frac{(1 - \cos x)}{(1 - \cos x)\cos x} = \sec x$$

3. a.  $2(1 - \sin^2 \theta) = 3 \sin \theta + 3$

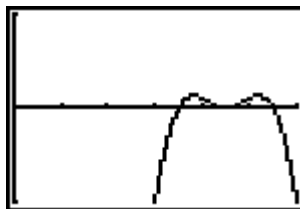
$$3 \sin \theta + 3 - 2 + 2 \sin^2 \theta = 0$$

$$2 \sin^2 \theta + 3 \sin \theta + 1 = 0$$

$$(2 \sin \theta + 1)(\sin \theta + 1) = 0$$

$$\sin \theta = -\frac{1}{2} \quad \parallel \quad \sin \theta = -1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \parallel \quad \frac{3\pi}{2}$$



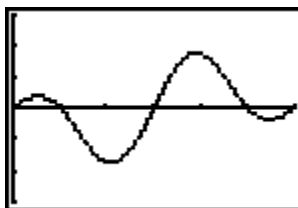
b.  $2 \cos \theta \sin \theta = \sin \theta$

$$2 \cos \theta \sin \theta - \sin \theta = 0$$

$$\sin \theta (2 \cos \theta - 1) = 0$$

$$\sin \theta = 0 \quad \parallel \quad \cos \theta = \frac{1}{2}$$

$$\theta = 0, \pi \quad \parallel \quad \frac{\pi}{3}, \frac{5\pi}{3}$$



c.  $2 \cos^2 \theta - 3 \cos \theta = 2$

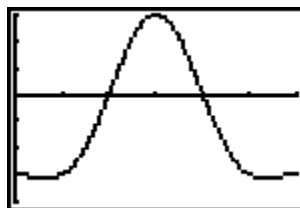
$$2 \cos^2 \theta - 3 \cos \theta - 2 = 0$$

$$(2 \cos \theta + 1)(\cos \theta - 2) = 0$$

$$\cos \theta = -\frac{1}{2} \quad \parallel \quad \cos \theta = 2$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \parallel \quad \text{None}$$

$$\theta = \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k$$



d. Same as b.

e.  $\cos 2\theta = -\cos \theta$

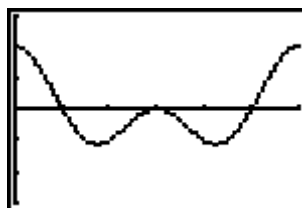
$$2 \cos^2 \theta - 1 = -\cos \theta$$

$$2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$\cos \theta = \frac{1}{2} \quad \parallel \quad \cos \theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \parallel \quad \pi$$



4. a.  $\cos(\sin^{-1} 2x) = \cos\theta$ , where  $\theta = \sin^{-1} 2x$  or  $\sin\theta = 2x$

$$\cos(\theta) = \sqrt{1 - (\sin\theta)^2} = \sqrt{1 - (2x)^2} = \boxed{\sqrt{1 - 4x^2}}$$

b.  $\cot(\tan^{-1} 2x) = \cot\theta$ , where  $\theta = \tan^{-1} 2x$  or  $\tan\theta = 2x$

$$\cot(\theta) = \frac{1}{\tan\theta} = \boxed{\frac{1}{2x}}$$

c.  $\cos(\cos^{-1} x + \sin^{-1} x) = \cos(\alpha + \beta)$ , where  $\alpha = \cos^{-1} x$ ,  $\beta = \sin^{-1} x$

$$\cos\alpha = x, \quad \sin\beta = x$$

$$\sin\alpha = \sqrt{1 - x^2}, \quad \cos\beta = \sqrt{1 - x^2}$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$= x\sqrt{1 - x^2} - (\sqrt{1 - x^2})x$$

$$= \boxed{0}$$

d.  $\cos(2 \sin^{-1} x) = \cos\theta$ , where  $\theta = \sin^{-1} x$  or  $\sin\theta = x$

$$\cos(\theta) = 1 - 2 \sin^2\theta = \boxed{1 - 2x^2}$$

5. A  $\sin(x - y) \cos y + \cos(x - y) \sin y = \sin((x - y) + y) = \sin x$