Evanescent waves and Total internal reflection

Under which type of reflection can total internal reflection occur: internal or external? What do those two terms mean?

What do you think happens to the wave at the interface when the wave undergoes total internal reflection? Does it penetrate some distance through the barrier or does it simply end at the barrier. Explain why you think this is the case.

Draw a diagram showing an incident ray and a transmitted ray. Label the wave vector in each region ($\mathbf{k}_i$ for incident and $\mathbf{k}_t$ for transmitted). The wave is in the x-y plane incident upon a interface in the x-z plane. z is directed out of the page. Label the angles $\theta_i$ and $\theta_t$ where these angles are defined in terms of the surface normal. Write down a relationship between these two angles.

Consider the transmitted wave of the form $\hat{E}_T = \hat{E}_{T0} e^{i\mathbf{k}_t \cdot \mathbf{r}} e^{-i \theta_t}$. Write an expression for $\mathbf{k}_t \cdot \mathbf{r}$ in terms of Cartesian coordinates and $\theta_t$. Let $\mathbf{r} = x \hat{x} + y \hat{y} + z \hat{z}$. Recall that the wave is limited to the x-y plane.
Now, rewrite your equation in terms of $\theta_i$ (this will require the use of the ratio of the indices of refraction).

Under the conditions of total internal reflection, how does $\sin(\theta_i)$ compare to the ratio of transmitted to incident indices of refraction? What does this mean about your expression for $\vec{k}' \cdot \vec{r}$?

Rewrite your expression for the transmitted wave as exponentials but in terms of Cartesian coordinates. Recall, that $e^{i(x+y+z)} = e^{ix}e^{iy}e^{iz}$. Separate the time, $x$, and $y$ parts of the expression.

What does this expression tell you about the wave at the interface? Consider in particular the term involving $y$. Explain.

Consider the situation of having two right angle prisms separated by some small thickness $t$. As you adjust this separation, what will happen? Does it depend upon the wavelength of the incident light?