1. It is possible that one can have polarized light that is not linearly polarized. This is called elliptical polarization. Suppose a beam of light was traveling in the $+\hat{z}$ direction and the electric field had the following form...

$$\vec{E} = (A\hat{x} + B\hat{y}) \exp[i(kz - \omega t)]$$

where $A$ and $B$ are complex constants. (NOTE: this is a generalized form for elliptical polarization).

For the special case where $B = iA$ with $A$ = real number, the electric field becomes...

$$\vec{E} = A(\hat{x} + i\hat{y}) \exp[i(kz - \omega t)] = A\hat{x} \exp[i(kz - \omega t)] + A\hat{y} \exp[i(kz - \omega t + \frac{\pi}{2})].$$

(a) Plot the electric field’s projection on the x-y plane for different times

\[
t = 0, \frac{\pi}{4\omega}, \frac{3\pi}{4\omega}, \frac{5\pi}{4\omega}, \frac{7\pi}{4\omega}, \frac{2\pi}{\omega} \quad \text{at} \quad z = 0.
\]

(b) Plot the magnetic field’s projection on the x-y plane for different times

\[
t = 0, \frac{\pi}{4\omega}, \frac{3\pi}{4\omega}, \frac{5\pi}{4\omega}, \frac{7\pi}{4\omega}, \frac{2\pi}{\omega} \quad \text{at} \quad z = 0.
\]

2. Suppose we had a material that would retard the electric and magnetic fields of light differently for two different axes. As an example, we will consider what will happen to linearly polarized light as it passes through a material with this property. For this example, assume that the light has the following electric field

$$\vec{E} = E_0 \hat{x} \exp[i(kz - \omega t)]$$

as it enters the material.

(a) The component of the electric field along the axis defined by $\pm \hat{x}$ will be phase shifted by $\phi$ and will be polarized in the $\pm \hat{x}$ direction after the light exits the material. The component of the electric field along the axis defined by $\pm \hat{y}$ will be phase shifted by $(\phi + \pi)$ and will be polarized in the $\pm \hat{y}$ direction after the light exits the material. Describe the polarization before and after the light exits the material.

(b) The component of the electric field along the axis defined by $\pm \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$ will be phase shifted by $\phi$ and will be polarized in the $\pm \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$ direction after the light exits the material. The component of the electric field along the axis defined by $\pm \frac{1}{\sqrt{2}}(-\hat{x} + \hat{y})$ will be phase shifted by $(\phi + \pi)$ and will be polarized in the $\pm \frac{1}{\sqrt{2}}(-\hat{x} + \hat{y})$ direction after the light exits the material. Describe the polarization before and after the light exits the material.

(c) The component of the electric field along the axis defined by $\pm (\cos\theta\hat{x} + \sin\theta\hat{y})$ will be phase shifted by $\phi$ and will be polarized in the $\pm (\cos\theta\hat{x} + \sin\theta\hat{y})$ direction after the light exits the material. The component of the electric field along the axis defined by $\pm (-\sin\theta\hat{x} + \cos\theta\hat{y})$ will be phase shifted by $(\phi + \pi)$ and will be polarized in the $\pm (-\sin\theta\hat{x} + \cos\theta\hat{y})$ direction after the light exits the material. Describe the polarization before and after the light exits the material.
**Lab 9**

In today’s investigation, you will exclusively use a polarized Helium – Neon (He-Ne) laser. The He-Ne lasers do not have the power stability of the semi-conductor lasers you previously used, but they are necessary since some of the optical components are highly wavelength sensitive. In order to get accurate measurements from the photometer, you should average the highest power and lower power you see over an approximate 15 second period.

Send the He-Ne laser beam through a polarizing beamsplitting cube (PBC) to determine the laser beam’s linear polarization. Describe the laser beam’s polarization. What is the ratio of transmitted light power through the cube vs. the reflected light power?

Put a linear polarizer in a plate rotator. Make sure you use the spanner wrench to tighten/loosen the locking ring. Then set up the equipment as shown below.

![Setup Diagram](image)

Rotate the linear polarizer to minimize the transmitted laser beam power. We will call this 0° and we will keep this definition throughout the rest of the investigation. NOTE: This designation more than likely does not agree with the values on the rotation stage.

Rotate the linear polarizer by iterations of 15° until you rotate 360°. Complete the table and then using Excel plot your results.

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Attach a half-wave plate ($\lambda/2$ – plate) to a plate rotator. Set up the equipment as shown below.

![Diagram](attachment:diagram.png)

Rotate the $\lambda/2$ – plate and describe what happens to the transmitted and reflected beams of light through the PBC as you turn the $\lambda/2$ – plate. What do you think the $\lambda/2$ – plate is doing to the polarization?

Set up the equipment as shown below.

![Diagram](attachment:diagram2.png)

First turn the linear polarizer to its previous zero location (where it minimized the light from the laser). Next, rotate the $\lambda/2$ – plate to minimized the light power at the photometer. We will call this 0° for the $\lambda/2$ – plate and we will keep this definition throughout the rest of the investigation. NOTE: This designation more than likely does not agree with the values on the rotation stage.

Predict how the light power reaching the photometer will change as a function of the linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. polarizer’s angle.
Explain your reasoning for the above sketch:

Rotate the linear polarizer by iterations of 15° until you rotate 360°. Complete the table and then using Excel plot your results (make sure that zero power is on the graph). Compare to your predictions. Resolve any differences.

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Next rotate the λ/2 – plate to 45°. Predict how the light power reaching the photometer will change as a function of the linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. the linear polarizer’s angle.

Explain your reasoning for the above sketch:
Rotate the linear polarizer by iterations of 15° until you rotate 360°. Complete the table and then using Excel plot your results (make sure that zero power is on the graph). Compare to your predictions. Resolve any differences.

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Next rotate the λ/2 – plate to 22°. Predict how the light power reaching the photometer will change as a function of the linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. the linear polarizer’s angle.

Explain your reasoning for the above sketch:

Rotate the linear polarizer by iterations of 15° until you rotate 360°. Complete the table and then using Excel plot your results (make sure that zero power is on the graph). Compare to your predictions. Resolve any differences.

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Recall question 2 on the pre-lab. From what you observed, how does a $\lambda/2$ – plate differ from the material described in question 2 of the pre-Lab?

For what reason is a $\lambda/2$ – plate called a half wave plate?

How do you think a $\lambda/2$ – plate effects the electric and magnetic fields associated with the light passing through the $\lambda/2$ – plate? How does the data from this investigation support your answer?

Attach a quarter-wave plate ($\lambda/4$ – plate) to a plate rotator. Set up the equipment as shown below.

![Diagram of laser, quarter-wave plate, and PBC]

Rotate the $\lambda/4$ – plate and describe what happens to the transmitted and reflected beams of light through the PBC as you turn the $\lambda/4$ – plate.

If light is polarized, one can always consider it a form of elliptical polarization. If one traces the electric field (or magnetic field) of a polarized electromagnetic wave about the propagation axis, it will trace out an ellipse (see problem 1 of the pre-lab). $\lambda/4$ – plates are useful for manipulating elliptical polarization.

For any ellipse, we can define it by 3 parameters: the semi-major axis $a$, the eccentricity $\epsilon$, and the tilt angle $\phi$. Eccentricity is the distance between the two foci divided by the semi-major axis. The eccentricity is zero for a perfect circle and one for a straight line. The eccentricity can also be found using the semi-minor axis $b$ (see the below sketch).
For elliptical polarization we can determine the tilt angle as well as the eccentricity by rotating a linear polarizer. The tilt angle is determined by angle at which the maximum amount of light passes through the linear polarizer. The eccentricity is found by

\[ \varepsilon = \frac{\sqrt{a^2 - b^2}}{a} \]

where \( \lambda/4 \) is the maximum signal through an ideal linear polarizer and \( \text{Min} \) is the minimum signal through an ideal linear polarizer.

Set up the equipment as shown below.

First turn the linear polarizer to its previous zero location (where it minimized the light from the laser). Next, rotate the \( \lambda/4 \) – plate to minimized the light power at the photometer. We will call this 0° for the \( \lambda/4 \) – plate and we will keep this definition throughout the rest of the investigation. NOTE: This designation more than likely does not agree with the values on the rotation stage.

Predict how the light power reaching the photometer will change as a function of the linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. the linear polarizer’s angle.
Explain your reasoning for the above sketch:

Rotate the linear polarizer by iterations of 15° until you rotate 360°. Complete the table and then using Excel plot your results (make sure that zero power is on the graph). Compare to your predictions. Resolve any differences.

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How is the light elliptically polarized? Draw a sketch that shows the eccentricity and the tilt angle of the elliptical polarization.
Rotate the $\lambda/4$ – plate by $22^\circ$. Predict how the light power reaching the photometer will change as a function of the linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. the linear polarizer’s angle.

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How is the light elliptically polarized? Draw a sketch that shows the eccentricity and the tilt angle of the elliptical polarization.
Set up the equipment as shown below.

First turn the linear polarizer to 90° (where it should allow the most light to pass through). Reflect the laser light back towards the laser so that you can see the returning laser spot just to the side of the emission hole. Next, rotate the \(\lambda/4\) – plate to minimize the returning light power. What is the angle of the \(\lambda/4\) – plate?

Remove the mirror and attach a second linear polarizer to a plate rotator. Put a piece of masking tape on the second linear polarizer’s rotator to distinguish it from the first linear polarizer. Rotate the second polarizer to minimize the light. We will call this angle (for the second linear polarizer) 0°.

Next, we switch the position of the \(\lambda/4\) – plate and the second linear polarizer
Predict how the light power reaching the photometer will change as a function of the second linear polarizer’s angle. That is, make a plot of the transmitted laser beam power vs. the linear polarizer’s angle.

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Recall question 2 on the pre-lab. From what you observed, how does a $\lambda/4$ – plate differ from the material described in question 2 of the pre-Lab?

For what reason is a $\lambda/4$ – plate called a quarter wave plate?

How do you think a $\lambda/4$ – plate effects the electric and magnetic fields associated with the light passing through the $\lambda/4$ – plate? How does the data from this investigation support your answer?