On Decomposition of Cartesian Products of Regular Graphs into Isomorphic Trees

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Ringel conjectured that for every tree \( T \) with \( m \) edges, the complete graph \( K_{2m+1} \) decomposes into copies of \( T \); such a decomposition is called a \( T \)-decomposition. Graham and Häggkvist conjectured that every \( 2m \)-regular graph has a \( T \)-decomposition and every \( m \)-regular bipartite graph has a \( T \)-decomposition. Snevily proved the the latter conjecture for some special classes by seeking more structure in the decompositions. Avgustinovich obtained results on decompositions of bipartite graphs into induced copies of \( T \) by considering labels on the edges of \( T \). We combine and extend these ideas to prove that the conjectures hold under various conditions for graphs that are cartesian products of regular graphs. This is joint work with Douglas B. West and Alexandr V. Kostochka.