



O.R. Applications

CO₂ emissions trading planning in combined heat and power production via multi-period stochastic optimization

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Abstract

The EU emissions trading scheme (ETS) taking effect in 2005 covers CO₂ emissions from specific large-scale industrial activities and combustion installations. A large number of existing and potential future combined heat and power (CHP) installations are subject to ETS and targeted for emissions reduction. CHP production is an important technology for efficient and clean provision of energy because of its superior carbon efficiency. The proper planning of emissions trading can help its potential into full play, making it become a true “winning technology” under ETS. Fuel mix or fuel switch will be the reasonable choices for fossil fuel based CHP producers to achieve their emissions targets at the lowest possible cost. In this paper we formulate CO₂ emissions trading planning of a CHP producer as a multi-period stochastic optimization problem and propose a stochastic simulation and coordination approach for considering the risk attitude of the producer, penalty for excessive emissions, and the confidence interval for emission estimates. In test runs with a realistic CHP production model, the proposed solution approach demonstrates good trading efficiency in terms of profit-to-turnover ratio. Considering the confidence interval for emission estimates can help the producer to reduce the transaction costs in emissions trading. Comparisons between fuel switch and fuel mix strategies show that fuel mix can provide good tradeoff between profit-making and emissions reduction.

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1. Introduction

Mitigation of the environmental impacts of energy production and use has become an integral part of energy policy planning. Consequently, the requirement for environmentally sound energy production

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technologies has gained much ground in the energy business. Recently the discussion has centered on the climate change. Combined heat and power (CHP) production is a leading technology to respond to the market demand and environmental concerns because of its high energy efficiency. The EU commission encourages the use of more efficient energy technologies, including CHP technology, producing fewer emissions per unit of output. Thus, the EU commission announces to raise the share of electricity produced by CHP technology from 9% to 18% during the years 1997–2010 (CEC Commission of the European Communities, 1997). An EU-wide emissions trading scheme (ETS) is also starting in 2005 to fulfill the EU commitment under the Climate Protection Protocol in Kyoto to cost-efficiently reduce the emissions of greenhouse gases by 8% during the period 2008–2012 compared with the 1990 level (Commission of the European Communities, 2000).

CHP production means the simultaneous production of useful heat and electric power. When steam or hot water is produced for an industrial plant or a residential area, electricity can be generated as a by-product. Vice versa, surplus heat from an electric power plant can be used for industrial purposes, or for heating space and water. CHP plants make the maximum use of the fuel's energy content by producing electricity and heat together with minimum wastage. The CHP plants can achieve a total efficiency of over 90%, while in conventional condensing power plants the efficiencies remain around 40%. Primary energy consumption in CHP production as compared with corresponding generation in separate processes is typically lowered by one third. The decrease in fuel consumption reduces the burden of energy production on the environment. That is, the CO₂ emissions are reduced at the same rate as the use of fuels is reduced. Moreover, a wide range of fuels can be used with modern CHP technology. Multi-fuel CHP plants can use, for example, solid fuel (coal, peat, and wood residues), liquid fuel (oil), gaseous fuel (natural gas) and even fuels with a low calorific value and high moisture content (waste, bio-fuel).

The ETS states that industrial activities that emit significant amounts of CO₂ must have a permit to do so. Such industries will be allocated allowances for specific amounts of greenhouse gas emissions for the relevant obligation periods based on national allocation plans of individual member countries. The individual producers can meet their compliance targets by reducing their emissions or by trading allowances within the EU. The producers must pay a penalty price for excessive emissions and have to make up the deficit by buying the lacking allowances in the beginning of the subsequent obligation period. Ideally, ETS will cause emissions to be reduced where it can be done most cost-efficiently.

The ETS provides both challenges and opportunities for the fossil fuel based energy sector, including CHP installations. A large number of existing and potential future CHP installations will be subject to ETS and targeted for emissions reduction. The high energy efficiency and low emissions make CHP production technologies environmentally friendly solutions compared with many other production forms. The flexibility in fuel choice facilitates fuel switch (change into fuels with lower specific CO₂ emissions) and fuel mix as reasonable alternatives for CHP producers to reduce their emissions. Evaluation of options for complying economically with the emissions target is complicated by many uncertainties involved in CHP production and emissions trading. In CHP, the heat and power production follows a joint characteristic, which means that the production planning of both commodities must be done in coordination. Under the deregulated electricity market, the power production should respond to the volatile spot price on the market, while heat must still be produced to balance the demand. In addition, fuel price and allowance price play an important role in fuel choice. Proper planning of emissions trading can help the potential of CHP production into full play, making CHP technology become a true “winning technology” under ETS.

Generally, emissions trading should be coordinated with other closely related operational decisions. Different emissions compliance options can also be employed in coordination. Under the US Clean Air Act Amendments (CAAA) of 1990, Lee et al. (1994) considered the coordination of SO₂ emissions trading with energy and spinning reserve transactions and consumption of take-or-pay fuels. They distributed adaptively the emissions target for the entire planning horizon into short-term operational targets, which were, in turn,

enforced in the associated unit commitment and dynamic dispatch subproblems. Manetsch (1994) proposed for long-term unit commitment and dispatch a method for integrating production planning with determination of SO₂ compliance options, such as switching into low sulfur coal and installing scrubbers. These papers emphasize the production planning of the power systems under the constraints of emissions control. Until now, most published papers that deal with CO₂ emissions trading in the energy sector are from the viewpoint of policy planning (Kunsh et al., 2004; Hauch, 2003; Söderholm and Strömberg, 2003). They do not address the trading problem itself.

In this paper, we study the CO₂ emissions trading planning problem of an individual CHP producer at the operational level. We formulate the CO₂ emissions trading planning of a CHP producer as a multi-period stochastic optimization problem and propose a solution approach that optimizes CHP plant operation and CO₂ emissions trading in coordination. During each trading period, the future CHP production until the end of the planning horizon is optimized based on scenarios for heat demand, power price and allowance price. Based on the optimized production plans the CO₂ emissions during the obligation period are estimated to determine how much allowances should be traded (bought or sold). The trading strategies are related to the risk attitude of the decision maker (DM). The proposed method can be used to evaluate the relative efficiency of different emission compliance options such as fuel switch and fuel mix. This paper extends the ideas presented in our early work (Rong et al., 2004) in five ways. Firstly, we explicitly consider the risk attitude of the DM in the problem formulation. Instead of maximizing the expected profit, we maximize the expected utility of the profit. Secondly, an hourly CHP production planning model replaces the previous aggregated model in production planning. Thirdly, we consider the transaction costs in the emissions trading and propose a trading strategy that depends on how uncertain the emission estimates are. Fourthly, we explicitly deal with the penalty for excessive emissions in the solution approach based on optimality conditions. Finally, we estimate the emissions using a weighted average with allowance prices as weights. Our early method did not apply weights in the estimation.

This paper is organized as follows. In Section 2, we describe the characteristics of CHP production and the uncertainties involved in the CHP production and emissions trading planning problem under the deregulated energy market. In Section 3, we formulate the CO₂ emissions trading planning problem for a CHP producer as a multi-period stochastic optimization problem. In Section 4, we present the solution approach for integrating CHP production planning and emissions trading and propose the corresponding trading strategies. In Section 5, we report the results on numerical experiments and compare the relative efficiency of the fuel switch and fuel mix strategies.

2. Characteristics of CHP production and uncertainties in emissions trading planning

2.1. Characteristics of CHP production

The primary concern of a CHP producer is to produce heat to satisfy variable demand. Normally heat production must meet the demand on an hourly basis. In CHP technology, heat and power production is linked together. The level of heat production determines the range in which the power generation can be adjusted and also the marginal cost function for power generation. A CHP plant can be represented by a joint characteristic that defines the dependency between production costs and heat and power generation as shown in Fig. 1.

Because the production costs are principally determined by the fuel consumption, the characteristic can alternatively specify the dependency between the fuel consumption and heat and power production. The characteristic can be either convex or non-convex. For the convex CHP plant, the characteristic operating region can be represented as a *convex combination* (see, e.g., Bazaraa and Shetty, 1993) of extreme points (c_j, p_j, q_j) , which are the corner points of the triangular facets in Fig. 1. A non-convex characteristic can be

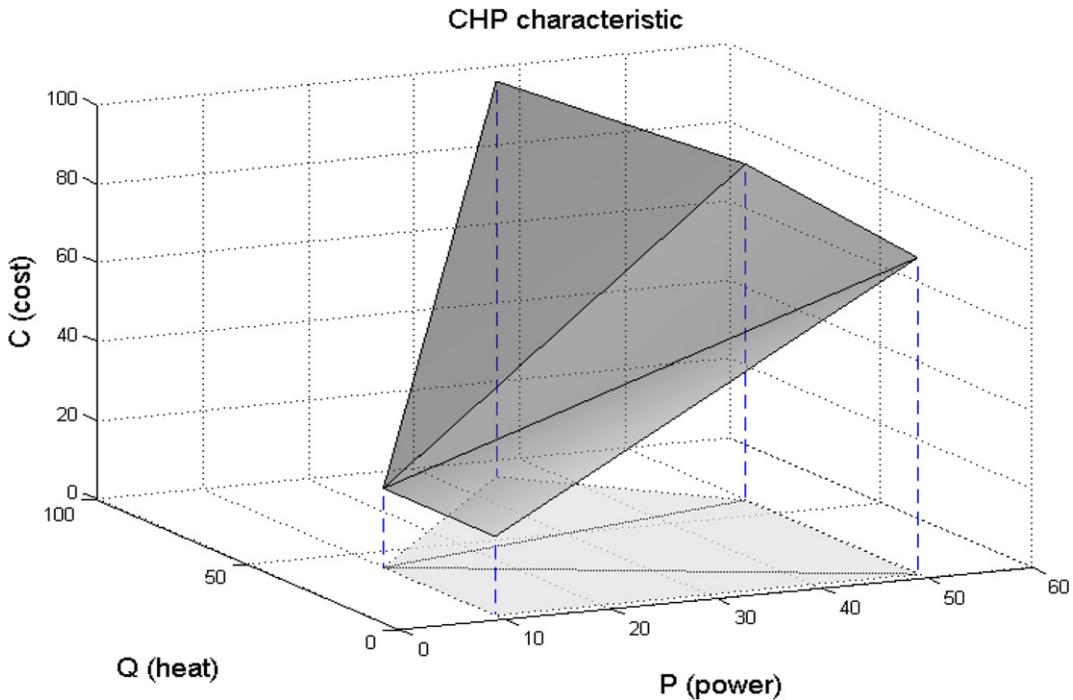


Fig. 1. Feasible operating region of a CHP plant.

divided into multiple convex sub-regions, which are encoded as alternative model components. The same modeling technique applies also to other energy acquisition components, such as separate heat and power plants, purchase contracts, and demand-side-management components. The interested reader can refer to [Lahdelma and Hakonen \(2003\)](#) for convex CHP plant modeling and to [Makkonen and Lahdelma \(2006\)](#) for non-convex modeling. On the liberalized power market, the rational producer should adjust power generation so that the marginal production cost equals the market price. As a result, the producer should optimize its heat and power production for each hour against the most recent forecasts for heat demand and spot market price for power.

CHP production planning is further complicated by the need to control the CO₂ emissions. The amount and type of consumed fuels determines the caused CO₂ emissions. Modern multi-fuel CHP plants are able to use different fuels and switch between fuels rapidly. In the fuel mix mode, several fuels can be used simultaneously within certain limits. The dispatch of fuels is governed by some rules. Generally the cheapest fuel is burned first unless there are other special requirements. The fuel prices and fuel mix affect the shape of CHP plant characteristic. The producer can adjust the production level, choose between different fuels, and trade emission allowances to balance its emissions with allowances. Thus, the CHP production planning problem must be solved in coordination with the emissions trading planning.

2.2. Uncertainties in integrated CHP production and emissions trading planning

In the planning problem we consider three main sources of uncertainty: heat demand, power price and allowance price. [Fig. 2](#) illustrates the uncertainties and their dependencies in the planning problem.

The heat demand depends almost entirely on local conditions. Municipal power plants generate mainly district heat. The uncertainty in district heating demand is almost entirely due to local weather conditions,

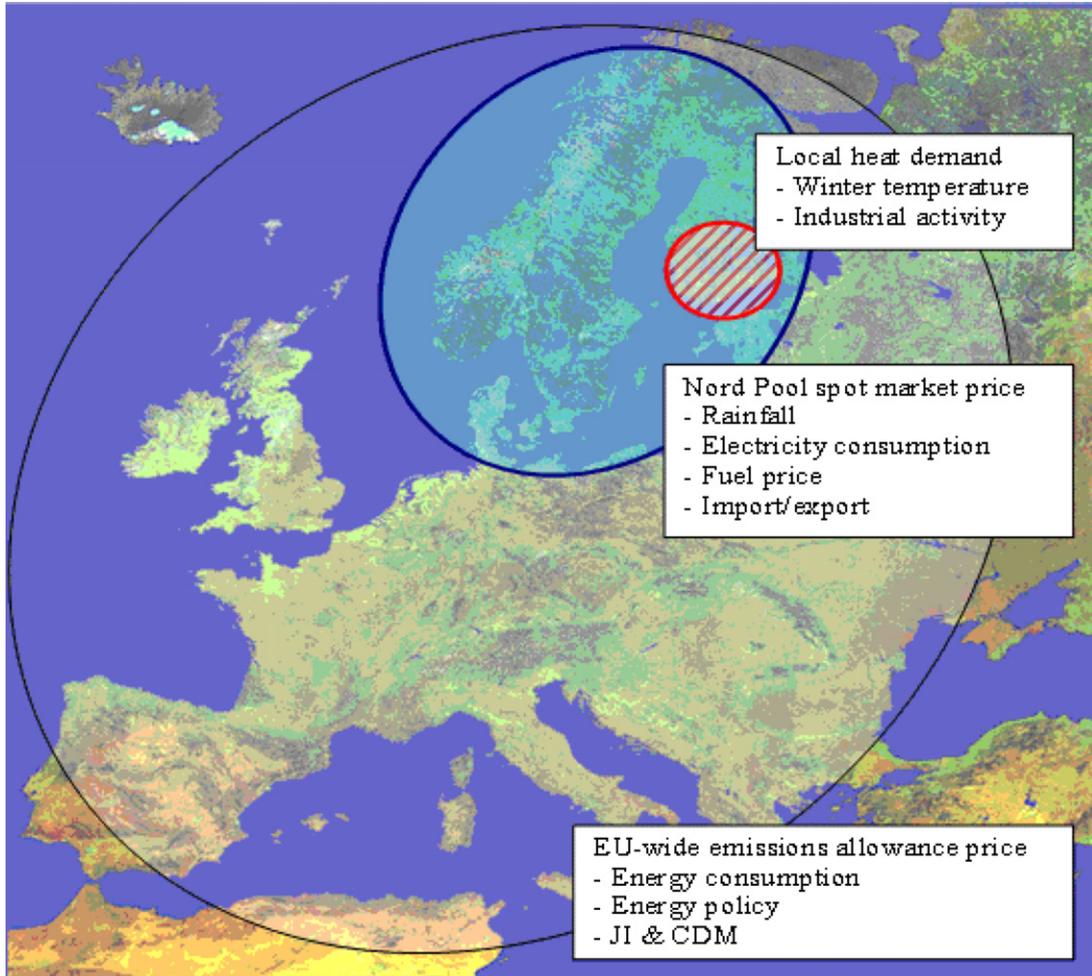


Fig. 2. Determination of heat demand, power price and emissions allowance price.

i.e., temperature, wind, etc. Industrial power plants generate process heat. This demand depends on how the industrial process is run.

The spot price for power is formed on the market as the equilibrium between power supply and demand. On the Nordic power market (Nord Pool, 2004) the most significant factor affecting the spot market price is the inflow to hydropower systems. Other important factors are seasonal variations, variations in fuel prices, producer decisions, consumer behavior, and import & export. The heat demand and power price are somewhat correlated. Cold weather will increase the demand for both heat and power, and consequently also the power price. However, this dependency is not very strong, because the heat demand is determined locally and power price on the entire market area.

The price of emission allowances will be determined by their supply and demand and the spot decisions of individual traders throughout EU. If all actors on the allowance market have the same information, the allowance price should all times reflect their common understanding about the future price development. This means that the spot price for allowances is the best possible estimate also for the future price. Factors

that affect the EU-wide allowance market are energy consumption, utilization of the Joint Implementation (JI) and Clean Development Mechanism (CDM), and political decisions about favoring and disfavoring some energy forms. The allowance costs will increase the marginal costs of the fossil fuel based power production, which makes allowance price and power price somewhat correlated.

3. Multi-period CHP production and emissions trading planning model

We assume that the DMs are risk averse and their preference structure in terms of the profit is represented by an increasing concave utility function $U(\cdot)$. The planning horizon is divided into periods $t = 1, \dots, \tau$ for CO₂ emissions trading. The trading periods can be, e.g., weeks or months. During each trading period, the emissions level can be affected by adjusting the fuel mix and power production level and balanced by emissions trading. Index $\tau + 1$ refers to the time after the planning horizon when the producers can still try to sell their surplus allowances and must make up for any deficit. The CHP production at different plants must be planned at much finer granularity than the emissions trading. For this reason each trading period is divided into hours $h \in H_t$. At the beginning of the planning horizon, the multi-period production and emissions trading planning model to maximize the expected utility of the profit can be stated as

$$\max E \left[U \left(\sum_{t=1}^{\tau} \sum_{h \in H_t} z^{pr}(\mathbf{x}_h, c_h^p) + \sum_{t=1}^{\tau} z^{tr}(f_t, c_t^f) + z^{tr}(f_{\tau+1}, c_{\tau+1}^f) \right) \right] \quad (1)$$

$$\text{s.t. } \langle \mathbf{x}, E \rangle \in \mathbf{X}(\mathbf{Q}), \quad (2)$$

$$\langle \mathbf{f}, E \rangle \in \mathbf{F}. \quad (3)$$

Here the function $z^{pr}(\cdot)$ is the net profit from hourly CHP production and $z^{tr}(\cdot)$ is the net profit from emissions trading during a trading period. \mathbf{x} , \mathbf{f} and E are variables in the model. The vector \mathbf{x} determines the CHP production in each hour, vector \mathbf{f} determines allowance trade in each trading period and scalar E is the cumulated emissions during the entire planning horizon. c_h^p , c_t^f and \mathbf{Q} are stochastic parameters. c_h^p is the hourly power price, c_t^f is the allowance price in period t and vector \mathbf{Q} contains the heat demand for each hour in the planning horizon. The set $\mathbf{X}(\mathbf{Q})$ represents the constraints of the CHP production process that depend on the heat demand. The set \mathbf{F} represents the constraints of the emissions trading process. To define the details of the model, we introduce the following notations.

Index sets

- B set of CHP plants and other supply or demand components modeled as CHP plants
- H, H_t set of hours in the planning horizon and in each trading period $t = 1, \dots, \tau$, correspondingly
- J, J_b set of extreme points of the characteristic operating region in all plants and in plant $b \in B$
- K set of fuels

Parameters

- δ ratio of allowance transaction costs to allowance price
- η_k specific CO₂ emission of fuel $k \in K$
- \mathbf{Q}, Q_h vector of heat demand during the planning horizon and the demand for hour $h \in H$
- $c^{F\#}$ penalty price for excessive emissions at the end of the planning horizon (period $\tau + 1$)
- c^{F+} emissions allowance purchase price at the end of the planning horizon, $c^{F+} = c_{\tau+1}^{f+}$
- c^{F-} emissions allowance sales price at the end of planning horizon, $c^{F-} = c_{\tau+1}^{f-}$
- c_t^f emissions allowance price in period $t = 1, \dots, \tau + 1$
- c_t^{f+} emissions allowance purchase price $c_t^{f+} = (1 + \delta)c_t^f$ for periods $t = 1, \dots, \tau$, including penalty for after last period $c_{\tau+1}^{f+} = (1 + \delta)c_{\tau+1}^f + c^{F\#}$

c_t^{f-}	emissions allowance sales price in period $t = 1, \dots, \tau + 1$, $c_t^{f-} = (1 - \delta)c_t^f$
c_h^p	power price in hour $h \in H$
c_h^q	heat price in hour $h \in H$
$c_{k,h}^r$	price of fuel $k \in K$ in hour $h \in H$
$p_{j,h}$	characteristic power coordinate $j \in J$ in hour $h \in H$
$q_{j,h}$	characteristic heat coordinate $j \in J$ in hour $h \in H$
$r_{k,j,h}$	consumption of fuel $k \in K$ at extreme point $j \in J$ in hour $h \in H$

Variables

\mathbf{f}, f_t	vector of allowance trade during the planning horizon and trade in trading period $t = 1, \dots, \tau + 1$
f_t^+, f_t^-	emissions allowance purchase and sales in trading period $t = 1, \dots, \tau + 1$
\mathbf{x}, \mathbf{x}_h	vector of decision variables determining the production during the planning horizon and in hour, $h \in H$, $\mathbf{x}_h = [x_{1,h}, \dots, x_{ J ,h}]$
$x_{j,h}$	contribution of extreme point $j \in J$ to CHP production in hour $h \in H$
x_h^p	power production in hour $h \in H$
E	cumulated emissions during the planning horizon
E^+, E^-	cumulated emissions exceeding and falling below the final allowance level F_τ
F_0, F_t	initial allocation and cumulated emissions allowance level at the end of trading period $t = 1, \dots, \tau$

3.1. The production model

The production model $\mathbf{X}(\mathbf{Q})$ determines the hourly production of heat and power as a sum of the production at different CHP plants and possible trade using various purchase and sales contracts. Each plant model consists of a sequence of hourly submodels that may be linked together by dynamic constraints, such as start-up and shut-down constraints, ramp constraints and storage constraints. The production model $\mathbf{X}(\mathbf{Q})$ can be subdivided into hourly models $\mathbf{X}_t(Q_t)$, which can be solved separately using suitable decomposition and coordination techniques. The applicable decomposition techniques depend on what kind of dynamic constraints are present. In this application we assume that no dynamic constraints are present and that the hourly plant models are convex. This means that we can solve the production model simply by solving the hourly models independently using a convex solver. The most efficient solvers for hourly convex CHP production planning problems are Power Simplex (PS) (Lahdelma and Hakonen, 2003), and the envelope-based algorithms ECON & ECOFF (Rong and Lahdelma, in press). PS has been implemented as part of the EHTO NEXUS energy optimization system (Lahdelma and Makkonen, 1996), which is in commercial use at several Finnish energy companies. Dynamic constraints and non-convex CHP models would require more sophisticated solution techniques for the production model, but would not affect the emissions trading model or the overall solution approach. Non-convex production planning problems can be solved, e.g., by using the Branch and Bound (BB) technique. Makkonen and Lahdelma (2006) solved non-convex planning problems by PS-based BB (PBB) and Rong and Lahdelma (2005c) developed envelope-based BB (EBB) algorithm for non-convex models. Rong and Lahdelma (2005b) analyzed the risks involved in CHP production expansion planning under the emissions trading scheme using the production model similar to that by Rong and Lahdelma (in press). The model by Rong and Lahdelma (in press) addresses the CHP production under the deregulated power market and the modeling technique is similar to that by Lahdelma and Hakonen (2003). Here we adopt a model that is similar to that by Rong and Lahdelma (in press). The hourly CHP production is modeled as a convex combination of characteristic extreme points for each hour $h \in H$:

$$\sum_{j \in J_b} x_{j,h} = 1, \quad b \in B, \tag{4}$$

$$\sum_{j \in J} p_{j,h} x_{j,h} - x_h^p = 0, \tag{5}$$

$$\sum_{j \in J} q_{j,h} x_{j,h} = Q_h, \tag{6}$$

$$x_{j,h} \geq 0, \quad j \in J. \tag{7}$$

In this formulation, the convex combination for each plant is encoded by a set of $x_{j,h}$ variables, indicating the operating level of each plant in terms of extreme points of the operating region, whose sum is one (4) and that are non-negative (7). The power balance (5) determines the net amount of power x_h^p that can be traded on the market at price c_h^p . The heat balance (6) states that the demand Q_h must be satisfied. The emissions during the planning horizon are the sum of the hourly emissions from the consumed fuels:

$$E = \sum_{h \in H} \sum_{j \in J} \left(\sum_{k \in K} \eta_k r_{k,j,h} \right) x_{j,h}. \tag{8}$$

The hourly profit from the CHP production is the power and heat sales revenue minus the production costs. The production costs are computed as a convex combination of fuel costs at the extreme points.

$$z^{pr}(\mathbf{x}_h, c_h^p) = c_h^p x_h^p + c_h^q Q_h - \sum_{j \in J} \left(\sum_{k \in K} c_{k,h}^r r_{k,j,h} \right) x_{j,h}. \tag{9}$$

3.2. The trading model

The trading model **F** determines the allowance trade in each trading period as follows:

$$F_t = F_{t-1} + f_t, \quad t = 1, \dots, \tau, \tag{10}$$

$$E = F_\tau + f_{\tau+1}, \tag{11}$$

$$f_t^+ = \max\{0, f_t\}, \quad t = 1, \dots, \tau + 1, \tag{12}$$

$$f_t^- = \max\{0, -f_t\}, \quad t = 1, \dots, \tau + 1. \tag{13}$$

$$E^+ = \max\{0, E - F_\tau\}, \tag{14}$$

$$E^- = \max\{0, F_\tau - E\}. \tag{15}$$

Constraints (10) determine the cumulated allowances at the end of each trading period. Constraint (11) requires that the emissions trading after the last period balances the emissions. Constraints (12) and (13) determine the amount of allowances bought and sold during each trading period. The combination of constraints (12) and (13) disallows the activity of buying and selling allowance to be done simultaneously. Constraints (14) and (15) determine the emissions exceeding and falling below the allowance level at the end of the planning horizon. The combination of constraints (14) and (15) implies that $E^+ E^- = 0$ and the results of allowance trading at the end of planning horizon take one of three results: exactly balance, fall below or exceed the realized emissions.

The trading profit during a period is either the revenue from selling or the negated costs of buying allowances. The purchase price after the end of the planning horizon (c^{F^+}) includes the penalty for excessive emissions.

$$z^r(f_t, c_t^f) = -c_t^{f+} f_t^+ + c_t^{f-} f_t^-, \quad t = 1, \dots, \tau, \tag{16}$$

$$z^r(f_{\tau+1}, c_{\tau+1}^f) = -c_{\tau+1}^{f+} f_{\tau+1}^+ + c_{\tau+1}^{f-} f_{\tau+1}^- = -c^{F+} E^+ + c^{F-} E^-. \tag{17}$$

4. Solution approach

We solve the production and trading problem using scenario analysis. Scenario analysis has proved to be an effective approach to address planning problems under uncertainty (Maranas et al., 1997; Mulvey and Shetty, 2004). We want to solve the production and trading planning model at each trading period $t^* \in \{1, \dots, \tau\}$ during the planning horizon. This means that for periods $1, \dots, t^* - 1$ the decision variables and stochastic parameters of the model (1)–(17) are fixed to their already realized values. Thus, optimization considers variation in the variables and stochastic parameters only for periods t^*, \dots, τ .

4.1. Scenario-based representation of the problem

The uncertainties of the operating environment were represented in the model (1)–(17) by stochastic parameters with some joint probability distribution. When considering the production and trading planning problem in the beginning of a trading period t^* , we approximate the future uncertainties by a set of scenarios. Each scenario defines a set of values for the stochastic parameters at each period in the future. The set of scenarios captures both the uncertainty of each stochastic parameter and the dependency information between them. To facilitate the representation, we extend the scenarios also for the past periods $1, \dots, t^* - 1$ to coincide with the realized history. In the current application the stochastic parameters are time series (vectors) for heat demand (\mathbf{Q}), power price (\mathbf{c}^p), and allowance price (\mathbf{c}^f). The heat demand and power price contain hourly values, but the allowance price contains weekly or monthly values.

To represent the scenario-based model formed at period t^* , we augment the previous notation by inserting both a scenario index s and period index t^* as the last two subscripts for the stochastic parameters and variables of the model. For example, $Q_h \rightarrow Q_{h,s,t^*}$ denotes the heat demand in hour h in scenario s generated at the beginning of trading period t^* . Let S denote the number of scenarios generated at each period. In the scenario representation the objective function to maximize the expected utility becomes

$$\begin{aligned} \max \frac{1}{S} \sum_{s=1}^S U \left(\sum_{t=1}^{\tau} \sum_{h \in H_t} \left(c_{h,s,t^*}^p x_{h,s,t^*}^p + c_h^q Q_{h,s,t^*} - \sum_{j \in J} \sum_{k \in K} c_{k,h}^r r_{k,j,h} x_{j,h,s,t^*} \right) \right. \\ \left. + \sum_{t=1}^{\tau} \left(-c_{t,s,t^*}^{f+} f_{t,s,t^*}^+ + c_{t,s,t^*}^{f-} f_{t,s,t^*}^- \right) - c_{s,t^*}^{F+} E_{s,t^*}^+ + c_{s,t^*}^{F-} E_{s,t^*}^- \right). \end{aligned} \tag{18}$$

The production and trading constraints for scenario s generated in period t^* become

$$\langle \mathbf{x}_{s,t^*}, E_{s,t^*} \rangle \in \mathbf{X}(\mathbf{Q}_{s,t^*}), \quad s = 1, \dots, S, \tag{19}$$

$$\langle \mathbf{f}_{s,t^*}, E_{s,t^*} \rangle \in \mathbf{F}, \quad s = 1, \dots, S. \tag{20}$$

However, solving (18)–(20) as a single problem is not meaningful, because it would allow the trading process to foresee the future in each scenario and yield infinite profit by speculative operations. This is of course not possible in practice. Instead, we must design an optimization scheme that can be implemented also in real life.

4.2. Decomposition and coordination approach

4.2.1. Decomposition of CHP production process and trading process

From formulas (18)–(20) we can see that the production and trading processes interact only through the emission variable E . No matter how the trading process is run, the production process must satisfy the heat demand and aim to maximize the production profit minus the emission costs. Recall that the spot price for allowances is the best estimate also for the future price. Therefore, when optimizing the hourly production, the spot price is the expected marginal cost for the caused emissions. Thus, the hourly CHP production should be optimized by introducing the allowance spot price as a penalty for the caused emissions. We add this penalty on the fuel prices based on their specific CO₂ emissions. The penalized fuel cost for fuel k in hour h for scenario s generated at the beginning of trading period t^* is then

$$\tilde{c}_{k,h,s,t^*}^r = c_{k,h}^r + c_{t,s,t^*}^f \eta_k, \quad h \in H_t. \tag{21}$$

After this, the CHP production for a scenario s generated at t^* can be optimized independently of the trading process:

$$\max \sum_{t=t^*}^{\tau} \sum_{h \in H_t} \left(c_{h,s,t^*}^p X_{h,s,t^*}^p + c_h^q Q_{h,s,t^*} - \sum_{j \in J} \sum_{k \in K} \tilde{c}_{k,h,s,t^*}^k r_{k,j,h} X_{j,h,s,t^*} \right) \tag{22}$$

$$\text{s.t. } \langle \mathbf{x}_{s,t^*}, E_{s,t^*} \rangle \in \mathbf{X}(\mathbf{Q}_{s,t^*}). \tag{23}$$

The solution of this problem combined with the realized history for periods $1, \dots, t^* - 1$ determines directly the cumulated emissions E_{s,t^*} and the production profit during the planning horizon in scenario s generated at period t^* .

4.2.2. Coordination between production and emissions trading processes

No matter how the production process is run, the trading process must balance the allowances with the caused emissions after the end of the planning horizon, and try to do it in the most cost-efficient way. In principle it is possible to trade allowances arbitrarily during the planning horizon. However, because the DM is risk-averse, we can restrict the trading process significantly. A risk-averse DM prefers a certain outcome to an uncertain outcome with the same expected value. Therefore the DM cannot expect to gain from buying or selling an excess of allowances in a speculative manner, because the current (known) allowance price is the best estimate also for the future (uncertain) price. If the future emissions are known accurately, but there is great uncertainty about the future allowance price, the DM should trade allowances early to meet the emissions and reduce the risk of having to pay a higher price. In contrast, if the future allowance price is known accurately, but there is great uncertainty about the future emissions, the DM should delay the trading in order to avoid the risk of aiming at the wrong target and having to re-balance the allowances again in subsequent trading periods. The latter case is particularly important when transaction costs are involved in the trading. In practice, we need a trading scheme that adapts simultaneously to different degrees of uncertainty both in the future allowance price and amount of caused emissions. Such a scheme will compromise between early and delayed trading to balance the allowances with the caused emissions.

The basic idea of the algorithm is to balance the allowances with the emissions that are estimated using the scenario-based production model. Because the cost of the emissions rather than the amount is relevant, we estimate the emissions weighted by the allowance price c_{t,s,t^*}^f in different periods and scenarios. This technique considers simultaneously the uncertainty both in the price and amount. To avoid selling and buying large quantities of allowances in subsequent periods due to fluctuations in emission estimates, we trade allowances to reach a confidence interval $[E_{t^*}^{\text{low}}, E_{t^*}^{\text{up}}]$ instead of meeting the (weighted) expected value $\mu(E_{t^*})$. The confidence interval can be determined either directly from the discrete set of scenarios, or based on a suitable probability distribution (such as the normal distribution) whose parameters are estimated

based on the scenarios. In Algorithm 1 below, we apply the latter technique. Here $\sigma(E_{t^*})$ denotes the weighted standard deviation of the cumulated emissions in scenarios during the planning horizon and the confidence factor n is the number of standard deviations that correspond to some confidence level $(1 - \alpha)$.

Algorithm 1. Heuristic for determining allowance trade in period t^* .

Step 1. Calculate the confidence limits for emission costs.

$$E_{t^*}^{\text{low}} = \mu(E_{t^*}) - n\sigma(E_{t^*}),$$

$$E_{t^*}^{\text{up}} = \mu(E_{t^*}) + n\sigma(E_{t^*}).$$

Step 2. Determine the target for the cumulated allowance level at the end of period t^* .

$$F_{t^*} = \max \{E_{t^*}^{\text{low}}, \min \{F_{t^*-1}, E_{t^*}^{\text{up}}\}\}.$$

Step 3. Determine the allowance trade f_{t^*} in period t^* .

$$f_{t^*} = F_{t^*} - F_{t^*-1}.$$

Considering the confidence interval for the emissions estimate is important when transaction costs are involved. The confidence factor determines the tradeoff between the trading frequency and quick reaction to variable allowance price. A small confidence factor will cause aggressive purchases and sales to follow random variations in the emission estimates. This can incur excessive transaction costs. However, a confidence factor that is too large will disable the trading activity. Generally, the confidence factor should be related to the transaction costs: the higher the transaction costs, the larger the confidence factor should be.

4.2.3. Dealing with penalty for excessive emissions

Trading in the last period τ is different from the previous periods, because this is the last opportunity to decide how to balance the allowances with the overall emissions. Trading after the last period is forced and depends on the stochastic outcome of the last period. Therefore, it will not be sufficient to aim at a confidence interval for the emissions. Instead, the producer should try to meet the emissions target as accurately as possible in different stochastic outcomes, taking into account the penalty for excessive emissions and the concave shape of the utility function. In the scenario representation this means that the producer should determine the trading level f_τ which maximizes the expected utility of the profit. By omitting the period index $t^* = \tau$ from the notations, we can rewrite the objective function (18) at the last period as

$$\max \bar{U}(f_\tau) = \frac{1}{S} \sum_{s=1}^S U(z_s(f_\tau)), \tag{24}$$

where

$$z_s(f_\tau) = Z_s^0 + c_{\tau,s}^{f^-} f_\tau^- - c_{\tau,s}^{f^+} f_\tau^+ + c_s^{F^-} E_s^- - c_s^{F^+} E_s^+ \tag{25}$$

is the overall profit in scenario s . Here Z_s^0 is the part of the profit that does not depend on f_τ , i.e., the already realized profit in the previous periods plus the production profit for the last period in scenario s . Let us examine the shape of the profit function. Based on (10)–(15) we can write the profit function as

$$z_s(f_\tau) = Z_s^0 + c_{\tau,s}^{f^-} \max\{0, -f_\tau\} - c_{\tau,s}^{f^+} \max\{0, f_\tau\} + c_s^{F^-} \max\{0, F_{\tau-1} + f_\tau - E_s\} - c_s^{F^+} \max\{0, E_s - F_{\tau-1} - f_\tau\}. \tag{26}$$

We can see that the profit in each scenario is a piecewise linear function with bending points at $f_\tau = 0$ and $f_\tau = E_s - F_{\tau-1}$. Furthermore, the profit function is concave, because the purchase price for allowances is

higher than the sales price, i.e., $c_{\tau,s}^{f^-} < c_{\tau,s}^{f^+}$ and $c_s^{F^-} < c_s^{F^+}$. Because the utility function is an increasing concave function, it means that also $U(z_s(f_\tau))$ is concave. Then also the expected utility (24) is a convex function of f_τ , because it is computed as an average over the scenario utilities. Assuming that the utility function is smooth (has a continuous derivative), then the expected utility function will be smooth except at the bending points ($f_\tau = 0$ and $f_\tau = E_s - F_{\tau-1}$).

The optimality conditions (Taha, 1992) state that the maximum of a concave, piecewise smooth function is either where the derivative becomes zero, or at a bending point where the derivative changes its sign. The derivative is obtained simply as the average of the derivatives of each scenario:

$$\lambda = d\bar{U}(f_\tau)/df = \frac{1}{S} \sum_{s=1}^S \lambda_s = 0, \tag{27}$$

where

$$\lambda_s = dU(z_s(f_\tau))/df_\tau. \tag{28}$$

The scenario-specific derivatives depend on the sign of the last period trade and whether the emissions exceed or fall below the allowances:

$$\lambda_s = U'(z_s(f_\tau))(c_s^{F^-} - c_{\tau,s}^{f^+}) \quad \text{when } f_\tau > 0, \quad E_s^- > 0, \tag{29}$$

$$\lambda_s = -U'(z_s(f_\tau))(c_{\tau,s}^{f^+} - c_s^{F^+}) \quad \text{when } f_\tau > 0, \quad E_s^+ > 0, \tag{30}$$

$$\lambda_s = -U'(z_s(f_\tau))(c_{\tau,s}^{f^-} - c_s^{F^-}) \quad \text{when } f_\tau < 0, \quad E_s^- > 0, \tag{31}$$

$$\lambda_s = U'(z_s(f_\tau))(c_s^{F^+} - c_{\tau,s}^{f^-}) \quad \text{when } f_\tau < 0, \quad E_s^+ > 0. \tag{32}$$

Here $U'(\cdot)$ is the derivative of $U(\cdot)$ with respect to z_s . Whether $\lambda(f_\tau)$ has a zero depends on the price coefficients in different scenarios. Normally the penalty for excessive emissions is large, which means that $c_s^{F^+} - c_{\tau,s}^{f^-} > 0$ in (32). Because $U'(\cdot)$ is positive, this means that $\lambda(f_\tau)$ is positive for large negative values of f_τ . If $\lambda(f_\tau)$ obtains negative values for $f_\tau \rightarrow \infty$ then there will be a zero in the range $f_\tau \in (-\infty, \infty)$. In the opposite case, the optimal solution is to buy an infinite amount of allowances in the last period at price $c_{\tau,s}^{f^+}$ and sell them at $c_s^{F^-}$ after the last period. This solution is not very likely to happen in reality, because it would require better information about the future allowance price than the other actors on the market have.

To guarantee a bounded solution and to avoid the speculative trading, we limit the value of f_τ between f_{\min} , which is the largest value making $E_s^+ \geq 0$ for all of generated scenarios, and f_{\max} , which is the smallest value ensuring $E_s^- \geq 0$ for all of generated scenarios. If $\lambda(f_\tau)$ does not change sign in that range, then we use the end point of the range as the solution. Otherwise we find the optimal solution to the last period trading problem by a modified binary search (Brassard and Bratley, 1996) algorithm. The binary search must consider discontinuities in $\lambda(f_\tau)$. The termination condition of the binary search must be relaxed to stop with a solution where a sufficiently narrow range for f_τ has been found. The algorithm for finding f_τ is presented below.

Algorithm 2. Finding the optimal solution with the penalty costs for excessive emissions.

Step 1. Determine the initial interval $[f_{\min}, f_{\max}]$ for the binary search.

$$f_{\max} = \max_s \{E_s - F_{\tau-1}\},$$

$$f_{\min} = \min_s \{E_s - F_{\tau-1}\}.$$

Step 2. Search the optimal allowance trade f_τ

if $(\lambda(f_{\max}) \geq 0)$

$f_\tau = f_{\max}$

else if $(\lambda(f_{\min}) \leq 0)$

$f_\tau = f_{\min}$

else

f_τ is found by binary search to satisfy either $\lambda(f_\tau) = 0$ or the left and right values for the binary search is close enough.

Step 3. Determine allowance level F_τ at the end of the planning horizon.

$F_\tau = F_{\tau-1} + f_\tau$.

4.3. Stochastic simulation and coordination algorithm

Now we summarize the solution approach for the integrated CHP production and emissions trading planning problem.

Fig. 3 illustrates the coordination between the production and trading processes in the algorithm. As time advances, the production process is run to satisfy variable heat demand and react to both variable power price and allowance price, which in turn, updates the emission estimates. In the trading process, allowance trade is determined to respond to the changed emission estimates and then the allowance level is updated. The specific procedures for the algorithm are given below. The notations used in the algorithm are the same as those in (1)–(20).

Algorithm 3. Stochastic simulation and coordination algorithm for the integrated CHP production and trading planning problem.

Step 1. Initialization: $F_0 = \langle \text{initial allowance allocation} \rangle$.

Step 2. Determine allowance trade f_{t^*} and cumulated allowance level F_{t^*} in each period t^* .

for $t^* \leftarrow 1$ to τ

Step 2.1. Generate scenarios (time series) $\{ \langle Q_{h,s,t^*}, c_{h,s,t^*}^p, c_{t,s,t^*}^f \rangle, s = 1, \dots, S \}$ spanning the planning horizon for stochastic parameters such as heat demand, power price, and emissions allowance price. (The values of parameters for $1, \dots, t^* - 1$ in scenarios coincide with realized history.)

Step 2.2. For each scenario, solve the CHP production model with penalized fuel price (21)–(23) and obtain caused emissions.

Step 2.3. Determine allowance trade f_{t^*} and cumulated allowance level F_{t^*}

if $(t^* < \tau)$

Determine f_{t^*} and F_{t^*} based on Algorithm 1.

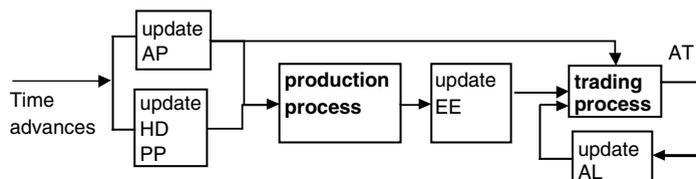


Fig. 3. Coordination between production process and trading process. AP: allowance price, HD: heat demand, PP: power price, EE: emission estimates, AL: allowance level, AT: allowance trade.

else

Determine f_{i^*} and F_{i^*} based on [Algorithm 2](#).

end for

Step 3. Check the balance between the emission level and allowance level at the end of the planning horizon.

$$E^+ = \max\{0, E - F_\tau\},$$

$$E^- = \max\{0, F_\tau - E\}.$$

Step 4. Compute the value of the objective function based on (1), (9), (16) and (17).

5. Computational results

Next we test numerically the proposed algorithm with weighted emissions estimation (WEA). We want to reach three goals with the test runs:

- (1) We assess the computational speed of the WEA algorithm.
- (2) We test the effectiveness of the WEA algorithm by comparing it against two simpler trading schemes.
- (3) We apply the WEA algorithm to evaluate the relative efficiency of the fuel switch and fuel mix strategies for fulfilling the emissions compliance.

5.1. Test problems

Since the major purpose of the numerical experiments is to test the performance of the proposed trading algorithm, we use a simple production model, as described in [Section 3](#). We assume that the production model is convex and no dynamic constraints are present in the system. The complexity of the production model mainly affects the computational speed of the algorithms. We will discuss this a little bit later.

We consider the planning problem of a CHP producer with power generation capacity around 100 MW. The hourly CHP plant model is based on a real-life power plant model and the characteristic extreme points of the CHP plant are generated based on different fuel choices as described in [Lahdelma and Rong \(2005\)](#). The number of extreme points in the plant varies typically from 5 to 20. In our example, the number of extreme points is 14 and 5, respectively, when the plant operates in the fuel mix and single-fuel modes. We apply a one-year planning horizon and divide it into 52 weekly trading periods.

We have generated six test problems based on the historical heat demand of a Finnish energy company in the past six years and historical power price in Nord Pool (The Nordic Power Exchange) ([Nord Pool, 2004](#)). We assume that the heat demand and power price vary around a time series forecast model according to a multivariate normal distribution. The forecast models and the variation are estimated based on history data. As no history information about allowance trade was available when this work was done, we generated the allowance price based on Brownian motion, varying from 5 to 25€/ton CO₂. This has turned out to be quite realistic, although even higher allowance prices have occurred during the year 2005. To simulate yearly trading, we generated 20 instances of each of the six test problems by sampling *history scenarios* from the assumed probability distributions. Within each history scenario and at each of the 52 periods, we generate 20 *future scenarios* to represent the future uncertainties. Thus a total of 1040 future planning problems are solved over the entire planning horizon while solving each test problem instance. The yearly trading simulation model is not only suitable for strategic analyses for the producer, but it serves also as a benchmark for the computational speed and effectiveness of the trading algorithm.

5.2. Computational speed

We have made the speed tests using the fuel mix model, which is a relatively complex model with a large number of extreme points. The production model was solved by Power Simplex (PS) (Lahdelma and Hakonen, 2003). To reduce the effect of random variations in CPU time measurements, each test problem was run 10 times and the average CPU time was computed. All test runs were performed in a 2.2 GHz Pentium 4 PC under the Windows XP operating system.

Table 1 gives the CPU time (seconds) for the yearly trading model by means of WEA as well as the time for solving a single instance of the yearly production model without trading. When solving the production planning model (22) and (23) at time t^* for one future scenario, we must in effect solve the production model for $(53 - t^*)$ weeks. During the solution of the yearly trading model (with 20 history scenarios and 20 future scenarios) we therefore must solve $20 \times 20 \times (52 + 51 + \dots + 1) = 551,200$ weekly models. The solution time for the yearly trading model should therefore be about $551,200/52 = 10,600$ times larger than for the yearly production model. From Table 1 we can compute the average ratio of 8816, which is quite close to the theoretically derived value. The slight advantage in the actual implementation is due to saved initialization overhead when solving a large number of similar problems.

By comparing the theoretical ratio with the empirical results, we can conclude that the computational speed of the proposed algorithm is principally determined by the time for solving the production model. Time performance of algorithms in more complex settings can be estimated based on the *time increase factor* observed in the test runs with comparable models. The time increase factor is the ratio between the solution times for the complex model and for the simple setting. For example, in non-convex planning, Makkonen and Lahdelma (2006) reported a time increase factor of 70, and for problems with dynamic energy storage constraints Lahdelma and Makkonen (1996) reported a time increase factor of 13. In a combined setting such time increase factors may be multiplied, resulting in solution times of 1 day for the non-convex yearly trading model with energy storage constraints. This is barely reasonable in strategic analyses using the PS algorithm. With the ECON/ECOFF (Rong and Lahdelma, in press) and EBB (Rong and Lahdelma, 2005c) algorithms we can expect much shorter solution times.

In on-line trading with the presented method, it is only necessary to solve at each period t^* the production model for the remaining part of the year using the 20 (or more) scenarios and to determine the weekly allowance trading according to Algorithm 1 or 2 (last period). In this setting the longest solution time will be 20 times the solution time for the yearly production model. This means that we can expect solution times of 0.2 seconds in the simplest setting and about 3 minutes in the combined complex setting.

5.3. Effectiveness of the algorithm

To test the effectiveness of the proposed WEA method, two comparisons are made. Firstly, we compare it against the direct (non-weighted) estimation algorithm (DEA) in our early work (Rong et al., 2004). DEA

Table 1
CPU time (seconds) for yearly trading by WEA and for solving single-yearly model

Model	Yearly trading	Yearly production model
A_1	102.0	0.0116
A_2	98.4	0.0111
A_3	102.7	0.0116
A_4	98.3	0.0113
A_5	97.6	0.0109
A_6	97.4	0.0112
Average	99.4	0.0113

uses caused emissions directly as an estimate for controlling the allowance level. Secondly, we compare WEA against an artificially deterministic trading algorithm (DTA). In DTA, the trading costs are estimated by $\bar{c}^f(1 \pm \delta)(E - F_0)$, where \bar{c}^f is the average allowance price over the entire planning horizon, δ is the ratio of the transaction costs to the allowance price, E is the caused emissions, and F_0 is the initial allowance allocation as described in Section 3. This comparison is to test the trading efficiency of WEA.

The objective function of the integrated CHP production and trading planning problem is to maximize the expected utility of the profit. The utility function plays an important role in characterizing the risk attitude of the DM. However, the specific value of utility function is not very meaningful. The following quantities are used to evaluate the performance of algorithm and the different emission compliance options (e.g., fuel switch and fuel mix).

- (a) Turnover (TO) consists of the revenue from selling the produced heat and power plus the value of the initial allowance allocation ($\bar{c}^f F_0$).
- (b) Certainty equivalence of profit (CEP) is the profit that corresponds to the expected utility that is maximized.
- (c) Profit-to-turnover ratio (PTTR) is $100 * CEP/TO$ %.

To evaluate the effectiveness of the algorithm, we compare the different algorithms in terms of different fuel choices at different transaction cost levels of emissions trading. We have three fuel choices and three transaction cost levels:

- FH fuel with higher specific CO₂ emissions (e.g., coal)
- FL fuel with lower specific CO₂ emissions (e.g., natural gas)
- FM fuel mix in which there are constraints on the maximum amount of high-emissive and low-emissive fuel

Three transaction cost levels are $\delta = 0$ (no transaction costs), 5% (moderate transaction costs) and 10% (high transaction costs) respectively. T_L denotes the transaction cost level in the subsequent tables reporting the results.

For effectiveness comparisons, we use the case with the higher emissions fuel as a reference. That is, the turnover of the fuel with higher emissions acts as the common denominator when the PTTR for different fuel choices are calculated.

The performance of WEA depends on the settings of the confidence interval in emission estimation. The confidence interval is determined by the confidence factor n , which is chosen heuristically. Generally, the higher the transaction cost level, the higher the confidence factor should be. Based on experiments, fuel choice also has some effect on n . Fuel with lower emissions (FL) reacts well to the higher confidence factor regardless of the transaction cost level. This may be explained by the penalized fuel price (21). For FL, the fuel price is higher and the specific CO₂ emission is lower. That means the emission penalty costs (related to allowance price) account for a relatively small portion in the penalized fuel price. Thus, FL is less sensitive to the allowance price as compared with the fuel with higher emissions (FH). As a result, a higher confidence factor is needed to guarantee a sufficient buffer even though when the transaction costs are lower. Table 2 shows the confidence factor settings for different fuel choices at different transaction cost levels.

Table 3 shows the PTTR difference between WEA and DTA for different fuel choices at different transaction cost levels. We can see that the improvement of WEA over DEA is significant. Three factors contribute to the improvement. Weighted emissions estimation combined with appropriate choice of confidence factors in estimation can suggest a more favorable volume of allowance trading based on the allowance price. Introduction of the optimization procedure in the last trading period can reduce the penalty costs for excessive emissions at the end of the planning horizon. Fig. 4 illustrates the effect of the

Table 2

Confidence factor settings for different fuel choices at different transaction cost levels

T_L	Fuel choices		
	FH	FL	FM
0%	0	3	0
5%	1	3	1
10%	1	3	2

Table 3

The PTTR difference between WEA and DEA for different fuel choices at different transaction cost levels (% points)

T_L	Problem	Fuel choice		
		FH	FL	FM
0%	A_1	1.07	0.54	0.45
	A_2	0.85	0.75	0.40
	A_3	1.06	0.57	0.26
	A_4	1.00	0.72	0.41
	A_5	1.91	2.09	0.87
	A_6	0.69	0.30	0.31
	Average	1.10	0.83	0.45
5%	A_1	2.59	1.17	2.24
	A_2	3.40	1.82	2.68
	A_3	2.71	1.60	2.05
	A_4	1.56	1.08	1.88
	A_5	2.79	2.56	2.71
	A_6	1.25	0.67	1.45
	Average	2.38	1.48	2.17
10%	A_1	4.38	1.80	4.72
	A_2	5.83	2.89	5.41
	A_3	4.83	2.63	4.66
	A_4	2.72	1.44	3.74
	A_5	4.05	3.03	5.40
	A_6	2.16	1.03	3.07
	Average	4.00	2.14	4.5

weighted estimation method and optimal procedures on the trading costs when there are no transaction costs. Fig. 5 illustrates the impact of confidence factors on the trading costs when transactions costs are involved. The relatively active trading activity for WEA in Fig. 4 implies that WEA can react better to uncertainty of allowance price than DEA. We also see the lower trading costs for WEA at the end of the planning horizon due to the adoption of optimal procedures. (The penalized trading costs of excessive emissions for WEA and DEA are 0.058 and 0.45 million €, respectively.) The total trading costs for WEA and DEA are 3.65 and 5.81 million €, respectively. Fig. 5 shows the profile of the trading process subject to transaction costs. With transaction costs, overly active trading activity is not encouraged. If the confidence factor is smaller (WEA0, $n = 0$), the trading frequency is too high, which implies higher transaction costs. On the other hand, if the confidence factor is higher (WEA2, $n = 2$), this reduces the trading activity and the process cannot react to allowance price variations well. In this example, WEA1 ($n = 1$) is an appropriate choice and provides a good tradeoff between trading frequency and the reaction to the uncertainty of allowance price. As a result, the total trading costs for WEA0, WEA1 and WEA2 are 6.04, 4.12 and 5.97 million €, respectively.

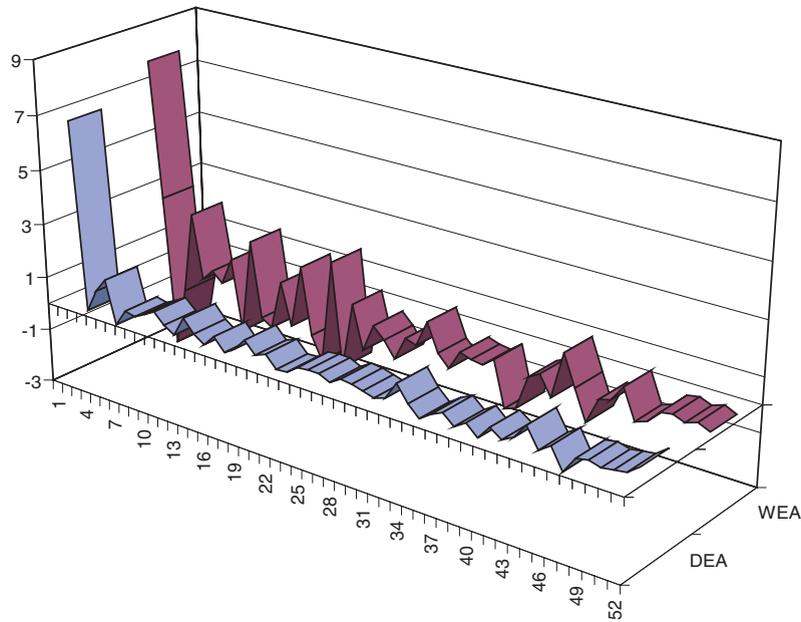


Fig. 4. Example of the effect of weighted estimation method and optimal procedures on the trading costs without transaction costs.

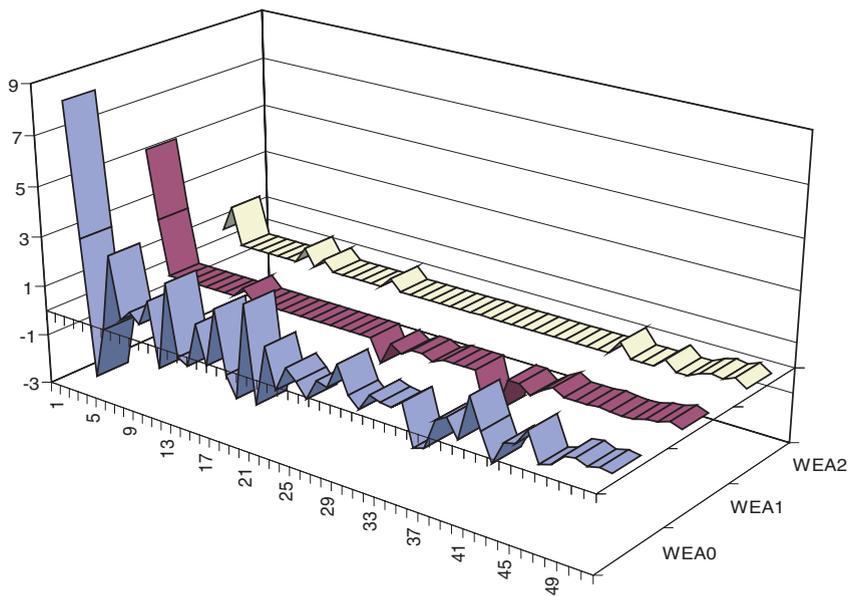


Fig. 5. Example of the effect of confidence factors on the trading costs with transaction costs.

As compared with DEA, WEA shows great increase in PTTR, especially when transaction costs for emissions trading are involved. Without transaction costs, the average improvement is 1.1%, 0.83% and 0.45% points for the FH, FL and FM strategies, respectively. When the transaction costs are moderate (5%), the average improvement is 2.38%, 1.48% and 2.17% points for FH, FL and FM.

costs are high (10%), the average improvement is 4%, 2.14% and 4.5% points for FH, FL and FM. The introduction of the confidence interval for emission estimates makes a major contribution in performance improvement for WEA when transaction costs are included, because the confidence interval can control the volume and frequency involved in trading more effectively.

Table 4 compares the performance of the proposed trading algorithm (WEA) with the artificial deterministic trading algorithm (DTA). We can see that WEA performs better than DTA, especially for fuel with higher emissions. This implies that the energy producers can benefit from emissions trading if they apply proper planning. For FH, the average improvement in PTTR is in range [4, 5.32] percentage points for different transaction costs. For FM, the average improvement is in range [1.32, 2.97] percentage points for different transaction costs. For FL, without transaction costs, the average improvement is 0.79% points. With transaction costs, the performance of WEA is close to that of DTA. This can again be explained by the fact that fuel with lower emissions is less sensitive to allowance price and emission estimation show less uncertainties.

Based on Tables 3 and 4, the improvement of WEA is significant in terms of PTTR. Since the turnover of the company involves a large monetary value, a small percentage point (e.g., 0.5) improvement in PTTR means a great increase in financial return.

5.4. Relative efficiency of fuel switch and fuel mix

Fuel switch means switching from the fuel with higher emissions into the fuel with lower emissions. Fuel mix means using a mixture of multiple fuels simultaneously. Here we consider the fuel mix with two fuels: one is the fuel with higher emissions (FH) and the other is the fuel with lower emissions (FL). The FL price is higher than FH price.

Table 4
The PTTR difference between WEA and DTA for different fuel choices at different transaction cost levels (% points)

T_L	Problem	Fuel choices		
		FH	FL	FM
0%	A_1	5.95	0.46	2.52
	A_2	5.30	1.18	2.91
	A_3	7.89	0.75	2.94
	A_4	3.17	0.48	2.37
	A_5	6.07	1.60	3.83
	A_6	3.54	0.26	1.92
	Average	5.32	0.79	2.79
5%	A_1	5.02	0.26	1.49
	A_2	4.56	0.73	2.10
	A_3	6.51	0.39	1.65
	A_4	2.38	-0.23	1.32
	A_5	5.40	0.84	2.63
	A_6	3.10	-0.17	1.14
	Average	4.50	0.30	1.77
10%	A_1	4.35	0.05	1.09
	A_2	3.67	0.28	1.72
	A_3	5.58	0.03	1.15
	A_4	2.21	-0.93	0.63
	A_5	5.11	0.08	2.28
	A_6	3.00	-0.60	0.84
	Average	3.99	-0.18	1.32

Based on the proposed algorithm WEA, we compare the relative efficiency of fuel switch and fuel mix for emissions trading for two situations. The FL price is higher in one situation and lower in the other situation. The transaction cost level for emissions trading is 5%. The reference situation is the fuel with higher emissions and FH price is fixed. Three criteria are considered: APTTR, REL and RTO.

APTTR absolute PTTR decrease (–) or increase (+) compared with the reference case
 REL relative emissions level decrease (–) or increase (+) compared with the reference case
 RTO relative turnover decrease (–) or increase (+) compared with the reference case

Tables 5 and 6 show the results for the lower and higher FL price respectively.

In terms of PTTR, fuel mix is the best for both lower and higher FL price based on Tables 5 and 6. This can be explained as follows. When the CHP plant operates in fuel mix mode, the fuels are dispatched based on the penalized fuel price (fuel price plus emission costs by the fuel) for different fuels. The fuel with the lowest penalized fuel price is used first. This situation can make use of the difference between allowance price and fuel price effectively to guarantee the economic dispatch. But if the plant operates in single-fuel mode, there is no advantage in fuel dispatch.

In terms of the emissions level, fuel mix lies between FL and FH regardless of the FL price. But the degree of emissions reduction varies with FL price. The emissions reduction is larger when FL price is lower and smaller when FL is higher because the share of FL fuel increases as the FL price decreases.

In terms of turnover, fuel mix lies between FL and FH regardless of the FL price. The turnover decreases as FL price increases because CHP should respond to market power price by adjusting the production level. That is, the decrease in turnover attributes to the decrease in the production level of electricity.

However, much risk is involved in fuel switch. Although fuel switch can make great contribution to emissions reduction, it may sacrifice both turnover and profit when FL price is higher. This situation makes fuel

Table 5
Performance of fuel switch and fuel mix for lower FL price

Problem	Switch			Mix		
	APTTR	REL	RTO	APTTR	REL	RTO
A_1	3.05	–52	2.43	3.61	–39	0.96
A_2	3.47	–53	2.09	4.14	–39	1.11
A_3	2.53	–52	2.82	3.15	–40	1.55
A_4	3.81	–53	0.91	4.70	–44	0.53
A_5	5.70	–53	1.10	6.11	–42	0.17
A_6	3.74	–54	0.13	4.67	–37	0.03
Average	3.72	–53	1.58	4.40	–40	0.72

Table 6
Performance of fuel switch and fuel mix for higher FL price

Problem	Switch			Mix		
	APTTR	REL	RTO	APTTR	REL	RTO
A_1	–4.42	–58	–4.63	–0.22	–23	–2.83
A_2	–4.37	–61	–6.84	0.14	–26	–4.19
A_3	–4.90	–59	–5.26	–0.81	–27	–3.23
A_4	–3.79	–56	–1.83	1.36	–18	–0.82
A_5	–5.17	–54	–0.34	0.72	–21	–0.48
A_6	–3.71	–57	–3.28	0.87	–24	–1.65
Average	–4.39	–57	–3.70	0.34	–23	–2.20

switch less attractive. When FL price is lower, fuel mix is comparable with fuel switch in terms of turnover and PTTR. Fuel switch shows some benefit in terms of turnover while fuel mix shows the equivalent benefit in terms of PTTR.

Overall, fuel mix can combat against variable fuel price and allowance price and reduce the risk. It can make good tradeoff between emissions reduction and profit-making. Maintaining the flexibility in fuel choice is the way to adapt to the emissions trading scheme.

6. Conclusions

The emissions trading scheme (ETS) poses both challenges and opportunities for the energy producers because the energy sector accounts for 30% of the CO₂ emissions in the EU. Important CO₂ reduction strategies rely on using fossil fuels more efficiently. Combined heat and power (CHP) production technology offers one of the biggest potentials for cost-efficient CO₂ reduction because of its superior carbon efficiency. On one hand, the ETS favors the technology for clean provision of energy. On the other hand, CHP installations should also be targeted for emissions reduction. The medium-term CO₂-abatement strategies will be a combination of the investment in new capacity and utilization of the existing capacity.

In this paper, we developed for a CHP producer an operational model and algorithm for planning simultaneously the emissions trading and emissions control through production and trading optimization. The algorithm is based on a stochastic simulation and coordination technique where the future uncertainties are represented by sets of scenarios. Stochastic simulation is an effective tool to deal with problems that involve many uncertainty factors. Successful applications include, e.g., risk analysis of energy trade (Makkonen and Lahdelma, 1998) and multi-criteria decision support in the deregulated energy market (Makkonen et al., 2003).

The algorithm proposed in this paper considers the risk attitude of the energy producer, the confidence interval in emission estimates and the penalty for excessive emissions. In test runs with a realistic CHP production model, real-life history data for heat demand and power price, and consensus estimates for the future allowance price, the proposed algorithm shows good trading efficiency in terms of profit-to-turnover ratio (PTTR) for different fuel choices at different transaction cost levels. It shows improvement over the previous trading strategy (Rong et al., 2004), especially for high transaction costs. It also shows improvement over an artificial deterministic trading strategy. This implies that the company can benefit from the proper planning of emissions trading. To minimize the risk, the company should take advantage of the opportunities that the emissions allowance market offers and take an active role in emissions trading because various fundamental factors such as fuel price and power price can affect the allowance price. The ongoing emissions trading has shown the volatility of emissions allowance prices (Climate Corporation, 2005). We also evaluated the relative efficiency of the fuel switch and fuel mix strategies based on the proposed algorithm. Fuel mix offers a good tradeoff between emissions reduction and profit-making. Changes of the energy market, increasing environmental awareness and novel energy production technologies create a need for new kinds of decision support tools for energy companies (Makkonen, 2005). The proposed algorithm is applicable also to new forms of clean energy provision technology such as trigeneration (Rong and Lahdelma, 2005a) and more extensive energy system planning such as multi-site CHP planning (Rong et al., 2006).

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