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The aim of this paper is to develop a mathematical model for a green product mix decision that incorporates capacity expansion features by using a mathematical programming approach. In order to satisfy customer orders when market demands exceed the company's production capacity, as well as maximise total profits, companies must study the feasibility of expanding capacity with regard to the production of parts. To place efficient controls on green product costs, we present the green product mix decision model for evaluating the benefits of expanding various types of capacity. By applying this model, companies that produce green products can make optimal decisions about further processing and capacity expansion.

Keywords: mathematical programming; green product mix decision; CO_2 emission cost; capacity expansion

1. Introduction

Global and regional emissions trading markets are emerging and drive strategic energy policy. The European Union Emission Trading Scheme (EU ETS) was a leading player and dominated carbon trading in 2006. This scheme is pivotal to achieving a truly 'global' carbon trading scheme and meeting the wider environmental challenge (Smale *et al.* 2006). There are two other Kyoto mechanisms: 'Joint Implementation' and 'Clean Development Mechanism (CDM)'. ETS is the largest greenhouse-gas market, but the scheme has been criticised on the grounds of over-allocation, violent price fluctuations and extravagant profits. The crucial development was the start of sulphur-dioxide trading in the US in 1995. That the sulphur-dioxide market was a success shaped how the Clinton Administration approached the negotiations that led to the 1997 Kyoto Protocol. In the Protocol, the industrialised nations promised that by Kyoto's 2008 to 2012 'commitment period' they would have limited their greenhouse-gas emissions to agreed proportions of their 1990 levels: 93% for the US and 92% for the European Community overall (MacKenzie 2009).

The EU ETS has regulated three targets. The first of these is to reduce the EU's CO_2 emissions by at least 20%; the second is to increase the proportion of renewable energies in its energy mix to 20%; and the third is to reduce its energy consumption by 20% by 2020. The improvement of energy efficiency is regarded as the fastest and most effective technique for reducing CO_2 emissions (IEA 2007). Energy conservation and CO_2 emission reduction potential can be defined in different ways, and the realisation of their potential is affected by various issues (Siitonen and Ahtila 2010). At the national or international level, energy conservation and CO_2 emission reduction potentials are typically evaluated on the basis of scenario studies. For example, the International Energy Agency (IEA) is using this methodology in the World Energy Outlook reports (IEA 2007). Moreover, the Intergovernmental Panel on Climate Change (IPCC) has developed emission situations for analysing the costs and benefits of different approaches to mitigating climate change (Metz *et al.* 2007). Energy efficiency plays a key role in CO_2 emission reduction across both IEA and IPCC situations.

The environmental impact of powered equipment is coming under close scrutiny. Energy consumed in manufacture and disposal is likely to be small in comparison with energy consumption over the typical 15-year life of such equipment. Thus, energy efficiency, along with the emission of gases which contribute to global warming, must take top priority when considering CO_2 emissions. Environmental improvements may have a significant impact on the reduction of CO_2 emissions. Several additional benefits come from improvements in energy efficiency, including reductions in the running cost of equipment and in primary fuel consumption, i.e. consumption of gas,

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oil and coal. However, these additional benefits are dependent on a number of different factors and can change according to circumstances (Hundy *et al.* 2008). MacKenzie (2009) analyses the development of carbon markets that trades in permits to emit greenhouse gases and credits earned by not emitting them. The 'cap' means that a maximum allowable aggregate total quantity of emissions exists. The 'trade' means that those for whom reductions are expensive will want to buy allowances rather than incurring disproportionate costs. The necessary supply of allowances is created by the financial incentive thereby provided to those who can make big cuts in emissions comparatively cheaply. The 'project-based' allowances concentrate on regulatory markets.

The increasingly worsening scale of the environmental problem is a critical threat to the development of society. High-pollution products are one of the main roots of environmental pollution in the manufacturing industry. Therefore, minimising the environmental impact of these products has become an important topic for all manufacturers (Sheng *et al.* 1995, Zust and Caduff 1997). Pursuing the manufacture of green products can be very beneficial in the alleviation of environmental burdens. Green products are those that have less of an impact on the environment or are less detrimental to human health than traditional equivalents. Green products might, typically, be formed or partially-formed from recycled components; manufactured in a more energy-conservative way; or supplied to the market with less packaging.

Products have become an important focus of environmental policy programmes. The common criticism about going green is that environmental practices add cost and reduce net returns. Nevertheless, the larger question is: can any business afford not to change its industrial colours? Boons (2002) presented the following six options for product chain management:

(1) Material reduction-reducing the amount of one or more materials necessary for producing the product.
(2) Material substitution-replacing one or more materials with alternatives that have less negative ecological effects.
(3) Material recycling-recycling a material which constitutes the product.
(4) Product substitution-replacing the product with another one which fulfils the same function.
(5) Product recycling-collecting and reusing the product.
(6) Eliminate function-stop fulfilling the function of the product.

As environmental pollution issues have become increasingly serious, environmental consciousness has become an important strategy for many business organisations. The product cost is defined as the cost of all the different components that make up the product. Product cost can then be optimised by determining the minimum value of costs that results from a given plant capacity and raw material cost (Stavropoulos and Zabaniotou 2009). Taking into account the complexity of such a techno-economic analysis, a useful suggestion could be to start the evaluations from a plant capacity corresponding to the break-even point, i.e. the capacity at which income equals production cost. Product-essential costs include machine hours, direct labour and direct material and may also include environmental protection costs, e.g. CO_2 emission costs.

Since the ETS started in 2005, the prices of allowances have varied between less than €1 and €30 per tonne of CO_2. There was a significant price crash in May 2005, which led to the hypothesis that electricity producers might use their market power to influence the prices of allowances (Jaehn and Letmathe 2010). Mirzaesmaeeli *et al.* (2010) developed a novel deterministic optimisation model that was applied to two case studies to examine the economic, structural and environmental effects that would result if the electricity sector was required to reduce its CO_2 emissions to a specified limit and minimise the overall cost of electricity. Fuel mixing or fuel switching will be the reasonable choices for fossil fuel-based combined heat and power producers to achieve their emissions targets at the lowest possible cost.

Kunsch *et al.* (2004) used system dynamics to validate the supply and demand sides in electricity markets and drew several notable conclusions. First, tax and permits have a common basis in the same optimisation mechanism of costs. Operators will act up to the level where marginal costs of reduction become equal to the tax, or the carbon price, on the whole market. Second, green certificates and CO_2 trading permits look like two different aspects of the same object. Finally, the tax and the green certificates are both charged to the demand side, while the permits are exchanged on the supply side. Take the underlying issue of a carbon market versus a carbon tax; many environmental activists prefer the latter, as do some economists, such as Nordhaus (2007). Likewise, the similarity of auctioning to a carbon tax; emissions markets seem almost always initially to involve free allocation because this approach reduces lobbying against them and political opposition. However, once markets are well-established, as the EU ETS now is, shifting to auctioning may become easier. Kunsch and Springael (2008) applied fuzzy reasoning techniques to a system dynamics model. They presented a planning and control methodology illustrated by a simplified case study on the carbon-tax design in the residential sector. From 2013 onwards, auctioning may be much more heavily employed, at least for sectors such as electricity that cannot in practice easily move production outside of the EU. So far, nevertheless, the political viability of a harmonised carbon tax, the obvious other route, remains questionable, because of the unanimity required.

According to the ETS, these regulations could include threshold values, penalties and taxes in addition to emission allowances that can be traded. Regarding the product mix and production quantities, Letmathe and Balakrishnan (2005) presented two mathematical models that can be used by firms to determine their optimal choice in the presence of several different types of environmental constraints, in addition to typical production constraints. Both mathematical models are comprehensive and incorporate several diverse production and environmental issues. The method of selecting green products of minimum cost under limited resources from the viewpoint of the manufacturer is quite important. As shown in Figure 1, machine cost, direct labour cost, material cost, product-level cost and CO_2 emission cost are present in the cost requirements of any green products needed by the manufacturer. However, each product may have different costs and, consequently, a different selling price and associated profit. Therefore, obtaining maximum total profits from green products under limited resources becomes important for the manufacturer.

The purpose of this paper is to incorporate capacity expansion features into the green product mix decision model by using a mathematical programming approach, and to examine the usefulness of green products and variable costs for production-related decisions subject to economies of scale. This analysis provides additional evidence of the conditions under which these costs lead to optimal resource allocation decisions. Section 2 describes a green product mix decision model with capacity expansion features. Section 3 uses a numerical example to demonstrate how to apply the model. Finally, the conclusion is presented in Section 4.

2. Green products mix decision model

A linear programming (LP) technique for a cost–volume–profit (CVP) model applied by Jaedicke (1961), called the 'product-mix' model in many management accounting or LP texts, could aid management in determining the optimal product mix, maximising total profit under some limits to or constraints on production or sales in the case of multi-product companies. In recent years, some researchers have utilised various mathematical programming approaches to conduct a product-mix decision analysis (Kee 1995; Malik & Sullivan 1995; Yahya-Zadel 1998; Kee & Schmidt 2000). Tsai and Hung (2009a) propose a fuzzy goal programming approach that integrates activity-based costing and performance evaluation in a value chain structure for optimal green

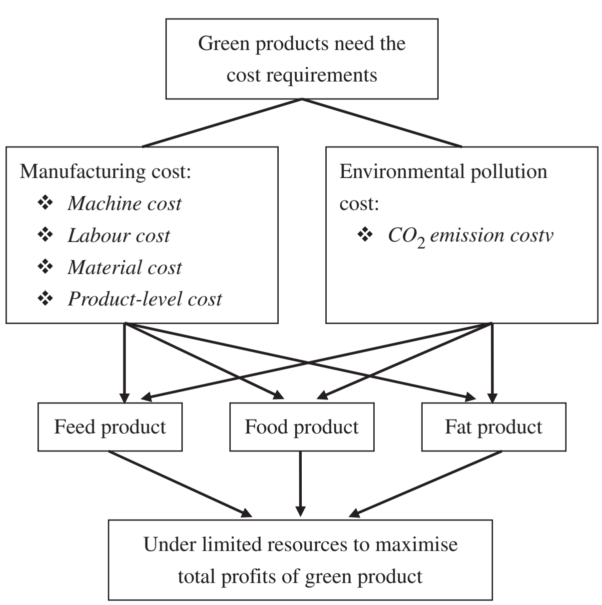

Figure 1. Green product framework.

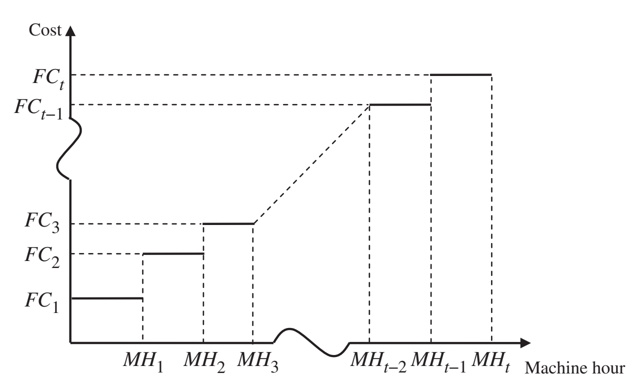

Figure 2. Stepwise machine cost.

supply chain supplier selection and flow allocation. Tsai and Hung (2009b) propose a two-stage multi-objective decision framework for the treatment and recycling system in waste electrical and electronic equipment (WEEE) reverse logistics management from an environmental supply chain perspective. Tsai *et al*. (2010) consider some factors, such as capacity expansion, management's degree of control over resources, purchase discount and change of product's price, to propose a general model. The model can help managers to make a product-mix decision and identify excess resources so that managers can redeploy them to optimise resource usage. In this paper, we will extend their research to incorporate capacity expansion features into the green product mix decision model.

2.1 *Assumptions*

In this paper, we develop the green product mix decision model, without loss of generality, assuming three green products, X_1, X_2 and X_3. In addition to the assumption relating to the production process, the green product mix decision model presented includes the following assumptions:

- The unit selling prices are constant within the relevant range.
- The specific process is regarded as a stepwise fixed cost that varies with machine hours.
- The machine hour resources can be expanded by renting or purchasing additional machines.
- The direct labour resources can be expanded by using overtime work or additional night shifts with higher wage rates.
- When the material quantity exceeds a threshold quantity, the purchaser receives a 10% discount for all material resources.
- CO_2 emissions are taxed at different rates, dependent on emission quantities, and the cost of CO_2 emissions is regarded as a piecewise variable cost.

2.2 *Capacity expansion features*

Product cost hierarchy, like capacity, has an important function in environmental goods. According to the assumptions described above, there are four capacity expansion features in the green product mix decision model.

Stepwise machine cost

As shown in Figure 2, the total machine cost cannot be traced to products with definite causal relationships, so we regard it as the common fixed cost and assume that its cost function is a stepwise function which varies with machine hours, observed from a prior cost behaviour analysis. The total machine cost is defined as FC_1 under the current capacity MH_1 machine hours. If the capacity is successively expanded to MH_2, $MH_3, \ldots MH_t$ machine hours, the total machine cost increases to FC_2, $FC_3, \ldots FC_t$, respectively. Let X_i be the production quantity of product i and δ_{ih} the requirement of machine hours for one unit of product i. As a result, the total machine cost and associated machine hour constraints are as follows (Tsai and Lin 1990):

$$\text{Total machine cost} = \sum_{k=0}^{t} FC_k\theta_k \tag{1}$$

Constraints:

$$\sum_{i=1}^{n} \delta_i X_i \leq \sum_{k=0}^{t} MH_k\theta_k \tag{2}$$

$$\sum_{k=0}^{t} \theta_k = 1 \tag{3}$$

where $(\theta_0, \theta_1, \dots \theta_t)$ is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero (Beale and Tomlin 1970; Williams 1985). When $\theta_k = 1$ $(k \neq 0)$, we know that the capacity needs to be expanded to the kth level, i.e. MH_k machine hours.

Piecewise direct labour cost and CO_2 emission cost

We have assumed that using overtime work or additional night shifts with higher wage rates can expand direct labour resources. Hence, the total direct labour cost function will be a piecewise linear function, as shown in Figure 3. The available normal direct labour hours are LH_1 and the direct labour hours can be expanded to LH_2 with the total direct labour cost being LC_1 and LC_2 at LH_1 and LH_2, respectively. For a result, the total direct labour cost and the associated constraints are as follows (Tsai and Lin 1990):

$$\text{Total direct labour cost} = LC_1\mu_1 + LC_2\mu_2 \tag{4}$$

Constraints:

$$\sum_{i=1}^{n} \alpha_i X_i - \sum_{\varphi=0}^{2} LH_\varphi \mu_\varphi \leq 0 \tag{5}$$

$$\text{TL} = LH_1\mu_1 + LH_2\mu_2 \tag{6}$$

$$\mu_0 - \lambda_1 \leq 0 \tag{7}$$

$$\mu_1 - \lambda_1 - \lambda_2 \leq 0 \tag{8}$$

$$\mu_2 - \lambda_2 \leq 0 \tag{9}$$

$$\mu_0 + \mu_1 + \mu_2 = 1 \tag{10}$$

$$\lambda_1 + \lambda_2 = 1 \tag{11}$$

where (λ_1, λ_2) is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero; (μ_0, μ_1, μ_2) is an SOS2 set of non-negative variables within which at most two adjacent variables, in the order given to the set, can be non-zero (Beale and Tomlin 1970; Williams 1985); TL is the total direct labour hours we need and its function depends on the case under study.

If $\lambda_1 = 1$, then $\lambda_2 = 0$ (from Equation (11)), $\mu_2 = 0$ (from Equation (9)), $\mu_0, \mu_1 \leq 1$ (from Equations (7) and (8)) and $\mu_0 + \mu_1 = 1$ (from Equation (10)). Therefore, from Equations (4) and (6) we know that total direct labour cost and total labour hours needed are $(LC_1\mu_1)$ and $(LH_1\mu_1)$, respectively; this means that we will not need the overtime work.

If $\lambda_2 = 1$, then $\lambda_1 = 0$ (from Equation (11)), $\mu_0 = 0$ (from Equation (7)), $\mu_1, \mu_2 \leq 1$ (from Equations (8) and (9)) and $\mu_1 + \mu_2 = 1$ (from Equation (10)). Therefore, from Equations (4) and (6) we know that total direct labour cost and total labour hours needed are $(LC_1\mu_1 + LC_2\mu_2)$ and $(LH_1\mu_1 + LH_2\mu_2)$, respectively; this means that we will need the overtime work.

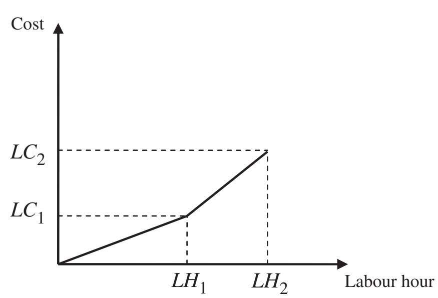

Figure 3. Piecewise direct labour cost.

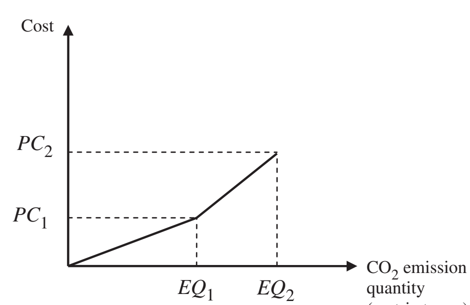

Figure 4. Piecewise CO_2 emission cost.

For each production activity of operational selection, the energy conservation and CO_2 emission reduction are calculated. Rong and Lahdelma (2007) formulate the CO_2 emissions trading planning of a combined heat and power producer as a multi-period possible optimisation problem. Rong and Lahdelma propose a stochastic imitation and coordination approach for considering the risk attitude of the producer, the penalty for excessive emissions and the confidence interval for emission estimates. The following definition of a CO_2 emission reduction equation was first presented by Möllersten *et al.* (2003):

$$TCO_2 = \sum_{i=1}^{n} (m_i + g_i + h_i) \tag{12}$$

where TCO_2 is the total quantity of CO_2 emission for product i, m_i is the change in CO_2 emissions from the mill, g_i is the change in CO_2 emissions from grid-based electricity production and h_i is the change in CO_2 emissions resulting from fuel (i.e. biomass) export from the mill.

A case explains the HFC-23 decomposition in MacKenzie's (2009) paper. Because the quantity of HFC-23 generated is affected by the precise parameters of the HCFC-22 production process, there is a need to reduce emission of the unnecessary greenhouse gas, HFC-23. Crucially, the allowable mass of HFC-23 that the measurement devices reveal has been decomposed is then multiplied by 11,700. Decomposing a tonne of HFC-23 means that an allowance to emit 11,700 tonnes of CO_2 is earned. The crucial figure, 11,700, is the product of a calculation of the 'global warming potential (GWP)' of HFC-23 published by the Intergovernmental Panel on Climate Change. GWP's canonical definition as following:

$$GWP = \frac{\int_G^{TP} \phi_x[x(t)]\mathrm{d}t}{\int_G^{TP} \phi_r[r(t)]\mathrm{d}t} \tag{13}$$

x designates the gas in question (e.g. HFC-23). ϕ_x is an estimate of the effect on the radiation balance at the tropopause (the boundary of the upper and lower atmosphere) of an increase in the amount of gas in the atmosphere, an effect measured in watts per square metre per kilogram. $x(t)$ is the mass of the gas that will remain in the atmosphere at time t from one kilogram released at time zero. TP is the overall time period in question: the calculation in the HFC-23 is commensurate with 100 years. The denominator is the equivalent integral for the reference gas, CO_2. $r(t)$ is not determined by releasing a kilogram of carbon dioxide and measuring what happens over a century, which is a mathematical function generated from a standard model (e.g. Siegenthaler and Joos 1992) of the exchange of carbon between the atmosphere, the oceans and the terrestrial biosphere.

We have assumed that CO_2 emissions are taxed at different rates depending on the quantity of emissions. Hence, as shown in Figure 4, the total CO_2 emission cost function will be a piecewise linear function. The standard CO_2 emission quantity is EQ_1 and the CO_2 emission quantity can be expanded to EQ_2; the total CO_2 emission costs are PC_1 and PC_2 at EQ_1 and EQ_2, respectively. As a result, the total CO_2 emission cost and the associated constraints are as follows (Tsai and Lin 1990):

$$\text{Total CO}_2\text{ emission cost} = PC_1\sigma_1 + PC_2\sigma_2 \tag{14}$$

Constraints:

$$\sum_{i=1}^{n} \beta_i X_i - \sum_{\varepsilon=0}^{2} EQ_\varepsilon \sigma_\varepsilon \leq 0 \tag{15}$$

$$\mathrm{TEQ} = EQ_1\sigma_1 + EQ_2\sigma_2 \tag{16}$$

$$\sigma_0 - \eta_1 \leq 0 \tag{17}$$

$$\sigma_1 - \eta_1 - \eta_2 \leq 0 \tag{18}$$

$$\sigma_2 - \eta_2 \leq 0 \tag{19}$$

$$\sigma_0 + \sigma_1 + \sigma_2 = 1 \tag{20}$$

$$\eta_1 + \eta_2 = 1 \tag{21}$$

where (η_1, η_2) is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero; $(\sigma_0, \sigma_1, \sigma_2)$ is an SOS2 set of non-negative variables within which at most two adjacent variables, in the order given to the set, can be non-zero (Beale and Tomlin 1970, Williams 1985); *TEQ* is the total CO_2 emission quantity we need and its function depends on the case under study.

If $\eta_1 = 1$, then $\eta_2 = 0$ (from Equation (21)), $\sigma_2 = 0$ (from Equation (19)), $\sigma_0, \sigma_1 \leq 1$ (from Equations (17) and (18)) and $\sigma_0 + \sigma_1 = 1$ (from Equation (20)). Therefore, from Equations (14) and (16) we know that total CO_2 emission cost and total CO_2 emission quantity needed are $(PC_1\sigma_1)$ and $(EQ_1\sigma_1)$, respectively; this means that we will not produce more than the emission standard.

If $\eta_2 = 1$, then $\eta_1 = 0$ (from Equation (21)), $\sigma_0 = 0$ (from Equation (17)), $\sigma_1, \sigma_2 \leq 1$ (from Equations (18) and (19)) and $\sigma_1 + \sigma_2 = 1$ (from Equation (20)). Therefore, from Equations (14) and (16) we know that total CO_2 emission cost and total CO_2 emission quantity needed are $(PC_1\sigma_1 + PC_2\sigma_2)$ and $(EQ_1\sigma_1 + EQ_2\sigma_2)$, respectively; this means that we will produce more than the emission standard.

Direct material cost

Next, we return to our assumption that purchasing material in large amounts can discount the price of the direct material resources. For example, the vendor of material allows a purchase discount of 10% for purchases that exceed M_r. Thus, the total material cost function will be a piecewise linear function, as shown in Figure 5. The available normal material quantity is M_r and the material quantity can be expanded to MD_r; the total material costs are l_r and lD_r at M_r and MD_r, respectively. The results of the total material cost and the associated constraints are then:

$$\text{Total material cost} = \sum_{r=1}^{s} l_r M_r + \sum_{r \in D} lD_r MD_r \tag{22}$$

Constraints:

$$\sum_{i=1}^{n} b_{ir} X_i \leq M_r, \quad r = 1, 2 \ldots, s, \tag{23}$$

$$M_r \leq W_r, \quad r = 1, 2 \ldots, s, \tag{24}$$

$$\sum_{i=1}^{n} b_{ir} X_i \leq M_r + MD_r, \quad r = 1, 2 \ldots, s, \tag{25}$$

$$MD_r \geq TD_r SD_r, \quad r = 1, 2 \ldots, s, \tag{26}$$

$$M_r < TD_r ND_r, \quad r = 1, 2 \ldots, s, \tag{27}$$

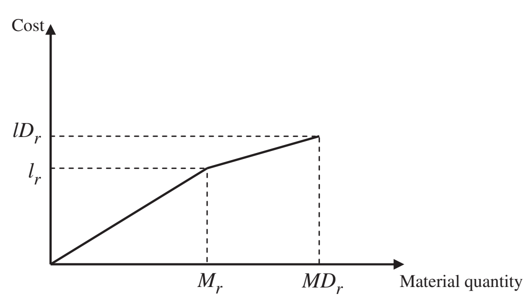

Figure 5. Piecewise direct material cost.

$$MD_r \leq W_r SD_r, \quad r = 1, 2 \ldots, s, \tag{28}$$

$$ND_r + SD_r = 1, \quad r = 1, 2 \ldots, s, \tag{29}$$

$$M_r \geq 0 \tag{30}$$

where (ND_r, SD_r) is an SOS1 set of 0–1 variables within which exactly one variable must be non-zero, and also denotes a purchase discount either ND_r = not taken or SD_r = taken; Equation (22) is the total direct material cost without and with a purchase discount, respectively.

Equations (23) to (30) are the constraints associated with the various types of material. For a material with a purchase discount condition, Equation (25) shows that the quantity of this material that either did or did not qualify for a purchase discount should satisfy the necessary amount demanded for producing each product. Equations (26) and (27) describe the conditions in which a purchase discount either did or did not qualify. Equation (28) sets a maximum quantity of a material with a purchase discount that can be ordered. And finally, Equation (29) ensures that one, and only one, of the conditions described by Equations (26) and (27) is in effect for each material.

Integrated cost models

The model for a green product mix decision with capacity expansions is as follows:

Maximise:

π = Total revenue – Total machine cost – Total direct labour cost – Total direct material cost – Total cost of environmental pollution – Product-level cost

$$\begin{aligned}&\sum_{i=1}^{n} P_i X_i - \sum_{k=0}^{t} FC_k \theta_k - (LC_1\mu_1 + LC_2\mu_2) - \left(\sum_{r=1}^{s} l_r M_r + \sum_{r\in D} lD_r MD_r\right) \\ &- (PC_1\sigma_1 + PC_2\sigma_2) - \sum_{i=1}^{n} g_i \rho_i R_i\end{aligned} \tag{31}$$

Subject to:

Machine hour constraints:

$$\sum_{i=1}^{n} \delta_i X_i - \sum_{k=0}^{t} MH_k \theta_k \leq 0 \tag{32}$$

$$\sum_{k=0}^{t} \theta_k = 1 \tag{33}$$

Direct labour constraints:

$$\sum_{i=1}^{n} \alpha_i X_i - \sum_{\varphi=0}^{2} LH_\varphi \mu_\varphi \leq 0 \tag{34}$$

$$\mathrm{TL} = LH_1\mu_1 + LH_2\mu_2 \tag{35}$$

$$\mu_0 - \lambda_1 \leq 0 \tag{36}$$

$$\mu_1 - \lambda_1 - \lambda_2 \leq 0 \tag{37}$$

$$\mu_2 - \lambda_2 \leq 0 \tag{38}$$

$$\mu_0 + \mu_1 + \mu_2 = 1 \tag{39}$$

$$\lambda_1 + \lambda_2 = 1 \tag{40}$$

CO_2 emission constraints:

$$\sum_{i=1}^{n} \beta_i X_i - \sum_{\varepsilon=0}^{2} EQ_\varepsilon \sigma_\varepsilon \le 0 \tag{41}$$

$$\text{TEQ} = EQ_1\sigma_1 + EQ_2\sigma_2 \tag{42}$$

$$\sigma_0 - \eta_1 \le 0 \tag{43}$$

$$\sigma_1 - \eta_1 - \eta_2 \le 0 \tag{44}$$

$$\sigma_2 - \eta_2 \le 0 \tag{45}$$

$$\sigma_0 + \sigma_1 + \sigma_2 = 1 \tag{46}$$

$$\eta_1 + \eta_2 = 1 \tag{47}$$

Direct material constraints:

$$\sum_{i=1}^{n} b_{ir} X_i \le M_r, \quad r = 1, 2 \ldots, s, \tag{48}$$

$$M_r \le W_r, \quad r = 1, 2 \ldots, s, \tag{49}$$

$$\sum_{i=1}^{n} b_{ir} X_i \le M_r + MD_r, \quad r = 1, 2 \ldots, s, \tag{50}$$

$$MD_r \ge TD_r SD_r, \quad r = 1, 2 \ldots, s, \tag{51}$$

$$M_r < TD_r ND_r, \quad r = 1, 2 \ldots, s, \tag{52}$$

$$MD_r \le W_r SD_r, \quad r = 1, 2 \ldots, s, \tag{53}$$

$$ND_r + SD_r = 1, \quad r = 1, 2 \ldots, s, \tag{54}$$

$$M_r \ge 0 \tag{55}$$

Product-level constraints:

$$X_i \le d_i R_i, \quad i = 1, 2 \ldots, n, \tag{56}$$

$$\sum_{i=1}^{n} \rho_i R_i \le V \tag{57}$$

$$X_i \ge 0, \quad i = 1, 2 \ldots, n, \tag{58}$$

$$(\mu_0, \mu_1, \mu_2; \sigma_0, \sigma_1, \sigma_2)\text{: An SOS2 set of non} - \text{negative variables} \tag{59}$$

$$(\lambda_1, \lambda_2; \eta_1, \eta_2; ND_r, SD_r)\text{: An SOS1 set of } 0 - 1 \text{ variables} \tag{60}$$

$$(\theta_0, \theta_1, \ldots \theta_t)\text{: An SOS1 set of } 0 - 1 \text{ variables} \tag{61}$$

$$R_i\text{: } 0 - 1 \text{ variables} \quad i = 1, 2, \ldots n \tag{62}$$

where

X_i	The production quantity of product i
P_i	The unit selling price of product i
δ_i	The requirement of machine hours for one unit of product i
α_i	The requirement of labour hours for one unit of product i
β_i	The CO_2 emission quantity for one unit of product i
ρ_i	The requirement of the product level for product i
l_r	The unit cost of the rth material without a purchase discount used
lD_r	The unit cost of the rth material with a purchase discount used
b_{ir}	The requirement of the rth material for one unit of product i
M_r	The quantities of the rth material without a purchase discount used
MD_r	The quantities of the rth material with a purchase discount used
TD_r	The threshold quantities of the rth material that an order must satisfy discount limit
W_r	The available quantity of the rth material
ND_r	A 0–1 variable. $ND_r = 1$ means that the quantities of the rth material dissatisfy the threshold of discount, otherwise, $ND_r = 0$
SD_r	A 0–1 variable. $SD_r = 1$ means that the quantities of the rth material satisfy the threshold of discount, otherwise, $SD_r = 0$
R_i	The indicator for producing product i ($R_i = 1$) or not producing product i ($R_i = 0$)
d_i	The maximum demand of product i
g_i	The actual designing cost per drawing for product i
V	The capacity limit of product level

Other variables and parameters are as mentioned before.

Equation (31) represents the total profit (π) function, and Equations (32) to (62) are the constraints associated with various resources. Equations (32) and (33) are the machine hour constraints. All equations relating to machine hours are described in the 'Stepwise machine cost' section. Equations (34) to (40) are the direct labour constraints; Equations (41) to (47) are CO_2 emission constraints. All equations related to direct labour and CO_2 emissions are described in the 'Piecewise direct labour cost and CO_2 emission cost' section. Equations (48) to (55) are the direct material constraints. All equations related to direct material are described in the 'Direct material cost' section. Equations (56) and (57) are associated with product-level constraints. Equation (56) is the market demand constraint; Equation (57) is the capacity constraint for product-level.

3. Numerical example

Take an example, company Y in the food industry (not identified for reasons of confidentiality), to illustrate how to apply the green decision model presented in this paper. We first obtain the optimal green product mix decision under current capacity. Then, we analyse the optimal green product mix decision with capacity expansions.

A food manufacturing company, Y, produces three main products: feed product ($i = 1$), food product ($i = 2$) and fat product ($i = 3$). Company Y needs to calculate the following essential costs in producing these three products: manufacturing costs (which may include machine, labour, material and product-level costs) and environmental cost (which may be the CO_2 emission cost). The related data for the numeric example are presented in Table 1.

From Table 1, we know that the total machine cost is \$35,400 under the current capacity $MH_1 = 23{,}600$ machine hours and that the capacity can be expanded to $MH_2 = 31{,}500$ or $MH_3 = 39{,}400$ machine hours by renting additional machines, with the total machine cost increasing to $FC_2 = \$51{,}990$ or $FC_3 = \$80{,}430$, respectively. Total CO_2 emission cost is \$125,000 under the current capacity $EQ_1 = 25{,}000$ metric tonnes, with the normal carbon rate of $T_1 = \$5$/tonne. The emission quantities may be expanded to $EQ_2 = 35{,}000$ metric tonnes, with the additional carbon rate of $T_2 = \$6$/tonne, with the total emission cost increasing to $PC_2 = \$185{,}000$. The available normal direct labour hours $LH_1 = 22{,}900$ hours, with the normal wage rate of $r_1 = \$4$/hr. The direct labour hours can be expanded to $LH_2 = 38{,}200$ hours, with the overtime wage rate of $r_2 = \$6$/hour. Further, the available normal direct material M_1 cost for each unit is $l_1 = \$5$/unit, and MD_1 cost for each unit is $lD_1 = \$4.5$/unit, including the purchase discount of 10%. The M_2 cost is $l_2 = \$3$/unit. Then, to obtain a purchase discount, the quantities of M_1 must satisfy $TD_1 = 36{,}000$. The available quantity of M_1 is $W_1 = 64{,}000$, and the available quantity of M_2 is $W_2 = 52{,}000$.

By using Equations (31) to (57), the green product mix decision model for the example is formulated as follows:
Maximise

$$\pi = 72X_1 + 55X_2 + 65X_3 - 35,400\theta_1 - 51,990\theta_2 - 80,430\theta_3 \\ - 91,600\mu_1 - 183,400\mu_2 - 5M_1 - 4.5MD_1 - 3M_2 \\ - 125,000\sigma_1 - 185,000\sigma_2 - 4,000R_1 - 2,500R_2 - 6,500R_3$$

Subject to:
Direct material constraints:

$$3X_1 + 2X_2 + 2X_3 - M_1 - MD_1 \leq 0$$

$$2X_1 + X_2 + 2X_3 - M_2 \leq 0$$

$$0 \leq M_1 < 36,000ND_1$$

$$MD_1 \leq 64,000SD_1$$

$$ND_1 + SD_1 = 1$$

$$0 \leq M_2 \leq 52,000$$

Machine constraints:

$$3X_1 + 2X_2 + 1.5X_3 - 23,600\theta_1 - 31,500\theta_2 - 39,400\theta_3 \leq 0$$

$$\theta_1 + \theta_2 + \theta_3 = 1$$

Table 1. Example data.

					Green product (i)			
					Feed product (i=1)	Food product (i=2)	Fat product (i=3)	Available capacity
Maximum demand				d_i	8,000	5,500	5,000	
Selling price				P_i	72	55	65	
Machining				δ_i	3	2	1.5	
Direct material constraint								
Cost/unit	$l_1 = \$5$/unit	$lD_1 = \$4.5$/unit	$TD_1 = 36{,}000$	b_{i1}	3	2	2	$W_1 = 64{,}000$
	$l_2 = \$3$/unit			b_{i2}	2	1	2	$W_2 = 52{,}000$
Product-level constraint								
Design	Drawings	$g_i =$ \$100		ρ_i	40	25	65	$V = 130$
Machine constraint								
Cost	$FC_1 = \$35{,}400$	$FC_2 = \$51{,}990$	$FC_3 = \$80{,}430$					
Machine hours	$MH_1 = 23{,}600$	$MH_2 = 31{,}500$	$MH_3 = 39{,}400$					
Direct labour constraint								
Cost	$LC_1 = \$91{,}600$	$LC_2 = \$183{,}400$		α_i	1	2	4	
Labour hours	$LH_1 = 22{,}900$	$LH_2 = 38{,}200$						
Wage rate	$r_1 = \$4$/hour	$r_2 = \$6$/hour						
CO_2 emission constraint								
Cost	$PC_1 = \$125{,}000$	$PC_2 = \$185{,}000$		β_i	2	1.5	3	
Emission quantities	$EQ_1 = 25{,}000$	$EQ_2 = 35{,}000$						
Tax rate	$T_1 = \$5$/tonne	$T_2 = \$6$/tonne						

Direct labour constraints:

$$X_1 + 2X_2 + 4X_3 - 22,900\mu_1 - 38,200\mu_2 \leq 0$$

$$\mu_0 - \lambda_1 \leq 0$$

$$\mu_1 - \lambda_1 - \lambda_2 \leq 0$$

$$\mu_2 - \lambda_2 \leq 0$$

$$\mu_0 + \mu_1 + \mu_2 = 1$$

$$\lambda_1 + \lambda_2 = 1$$

CO_2 emission constraints:

$$2X_1 + 1.5X_2 + 3X_3 - 25,000\sigma_1 - 35,000\sigma_2 \leq 0$$

$$\sigma_0 - \eta_1 \leq 0$$

$$\sigma_1 - \eta_1 - \eta_2 \leq 0$$

$$\sigma_2 - \eta_2 \leq 0$$

$$\sigma_0 + \sigma_1 + \sigma_2 = 1$$

$$\eta_1 + \eta_2 = 1$$

Product-level constraints:

$$X_1 - 8,000R_1 \leq 0$$

$$X_2 - 5,500R_2 \leq 0$$

$$X_3 - 5,000R_3 \leq 0$$

$$40R_1 + 25R_2 + 65R_3 \leq 130$$

where X_i, $i = 1,2,3$; $\mu_1, \mu_2, \mu_3, \sigma_1, \sigma_2, \sigma_3 \geq 0$; $\lambda_1, \lambda_2, \eta_1, \eta_2, ND_1, SD_1, \theta_k = 0, 1, k = 1, 2, 3$, $R_i, i = 1, 2, 3$. This is a mixed-integer programming (MIP) model. This problem is solved by utilising LINGO 8.0 software, and the following optimal solution is obtained:

$X_1 = 7\,514$	$X_2 = 5\,498$	$X_3 = 3\,908$
$\theta_1 = 0$	$\theta_2 = 0$	$\theta_3 = 1$
$\mu_0 = 0$	$\mu_1 = 0$	$\mu_2 = 1$
$M_1 = 0$	$MD_1 = 41{,}354$	$M_2 = 28{,}342$
$\sigma_0 = 0$	$\sigma_1 = 0$	$\sigma_2 = 1$
$R_1 = 1$	$R_2 = 1$	$R_3 = 1$
$\lambda_1 = 0$	$\lambda_2 = 1$	
$\eta_1 = 0$	$\eta_2 = 1$	
$ND_1 = 0$	$SD_1 = 1$	

Accordingly, the optimal green product mix is $(X_1, X_2, X_3) = (7{,}514,\ 5{,}498,\ 3{,}908)$, which requires 41,354 units $(= 3 \times 7,514 + 2 \times 5,498 + 2 \times 3,908)$ of the first type of material, 28,342 units $(= 2 \times 7,514 + 5,498 + 2 \times 3,908)$ of the second type of material, 39,400 units $(= 3 \times 7,514 + 2 \times 5,498 + 1.5 \times 3,908)$ machine hours, 34,142 units $(= 7,514 + 2 \times 5,498 + 4 \times 3,908)$ direct labour hours, and 34,999 units $(= 2 \times 7,514 + 1.5 \times 5,498 + 3 \times 3,908)$ CO_2 emission quantities. The total profit π is $364,469.

4. Conclusions

With environmental pollution becoming a more serious issue, many business organisations have realised the importance of environmental consciousness in their strategies. In a successful green product mix decision, the advantages gained from cost savings are an important factor under consideration. Therefore, decisions about the green product mix require an accurate analysis of related costs. We applied a mathematical programming model to green products to trace resource costs. This improved the accuracy of products cost (object) data derived from the traditional volume-based or unit-based costing systems. To extend the existing research literature, we incorporated capacity expansion features into the green product mix decision model by using a mathematical programming approach.

Firms that produce green products have to make decisions about processing and capacity expansion in order to maximise total profit. In this paper, a mathematical programming model for green product-related decisions and a numerical example were used to demonstrate how to apply the model under different conditions. Through accurate analysis of relevant costs, firms can maintain the equilibrium of internal production while also obtaining a competitive advantage by producing green products. Current traditional product mix decision models do not explicitly consider capacity expansions. This paper contributes to the management sciences and accounting literature by developing a new mixed integer programming green product mix model that maximises a firm's profit with the following major types of mathematical constraint: (1) stepwise machine hour constraints, (2) piecewise direct labour constraints, (3) direct material constraints, (4) piecewise CO_2 emission constraints, (5) product-level constraints. Using this model, we may evaluate the benefit of expanding various types of capacity simultaneously. By applying this model, companies that produce green products will be able to make optimal decisions about further capacity expansion.

In recent years, activity-based costing (ABC) has become a popular and useful cost management technique for both accounting academics and business practice. Through interactive developments in both academic and practical circles, ABC has been applied to various business functions and different industries. In future studies, researchers can incorporate ABC into the model and explore more complicated and realistic situations in green product mix decisions in relation to capacity expansion.

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