Homework Solutions
Chapter 11

(4) In this simple mass-density problem, what we're really being asked for is the radius of a brass sphere whose weight is 120 N. Newton's second law, the definition of mass density, and the mensuration (relating to measurement) formula for the volume of a sphere are involved here.

If the sphere's weight, *W*, is known, we can calculate its mass:

$$W = mg \quad \Rightarrow \quad m = \frac{W}{g} = \frac{120\ \text{N}}{9.8\ \text{m/s}^2} = 12.25\ \text{kg}$$

The sphere's volume is related to both its radius and its mass:

$$V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad r = \sqrt[3]{\frac{3V}{4\pi}} \qquad \rho = \frac{m}{V} \quad \Rightarrow \quad V = \frac{m}{\rho} = \frac{12.25\ \text{kg}}{8470\ \text{kg/m}^3} = 1.446\times10^{-3}\ \text{m}^3$$

The mass density was obtained from Table 11.1 on p. 322 of your textbook. Substituting this value of the volume:

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3\cdot1.446\times10^{-3}\ \text{m}^3}{4\pi}} = 7.0\times10^{-2}\ \text{m}$$

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(9) Let ϕ_G be the volume fraction of ethylene glycol in the solution
Let ρ_S be the solution's mass density
Let ρ_W be the mass density of water
Let ρ_G be the mass density of ethylene glycol

Then:

$$\rho_s = \phi_G\rho_G + (1 - \phi_G)\rho_W = \phi_G\rho_G + \rho_W - \phi_G\rho_W$$

$$\rho_S - \rho_W = \phi_G(\rho_G - \rho_W)$$

$$\varphi_G = \frac{\rho_S - \rho_W}{\rho_G - \rho_W}$$

Since the mass density of the solution is its specific gravity times the mass density of water, we can substitute:

$$\phi_G = \frac{SG\rho_W - \rho_W}{\rho_G - \rho_W} = \frac{\rho_W\left(SG-1\right)}{\rho_G - \rho_W} = \frac{\left(1000\text{ kg/m}^3\right)\left(1.073\text{ - }1\right)}{1116\text{ kg/m}^3\text{ - }1000\text{ kg/m}^3} = 0.629 = 62.9\%$$

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(13) The normal force exerted upward by the ground on each tire is half the combined weight of the bicycle and rider (since the problem tells us to assume that the two tires are equally loaded). The normal force on each tire is

$$N = \frac{625\text{ N} + 98\text{ N}}{2} = 361.5\text{ N}$$

The equal force exerted downward on the ground by the tire is the pressure in the tire times the area of contact:

$F = N = PA$. Solving for A:

$$A = \frac{N}{P} = \frac{361.5\text{ N}}{7.60\times10^5\text{ Pa}} = 4.76\times10^{-4}\text{ m}^2$$

The fact that the specified pressure is "gauge pressure" (that is, pressure in excess of atmospheric pressure) means that we need not concern ourselves here with the equilibrium between the outward force exerted on the tire by the (absolute) pressure within, and the inward force exerted by atmospheric pressure.

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(17) The pressure due to the cylinder is: $P_{cyl} = \frac{\pi r^2 h\rho g}{\pi r^2} = h\rho g$

The pressure due to the hemisphere: $P_{hemi} = \frac{\frac{2}{3}\pi r^3\rho g}{\pi r^2} = \frac{2}{3}r\rho g$

Equating the two and solving for r gives

$$\frac{2}{3}r\rho g = h\rho g$$

$$r = \frac{3h}{2} = \frac{3\left(0.500\text{ m}\right)}{2} = 0.750\text{ m}$$

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(23) This was really too simple a problem to assign, I suppose, but I couldn't resist: ooooh, neat, a dinosaur problem! After the interesting discussion of the possibilities concerning "Barosaurus" in the problem, to solve it seems a bit anticlimactic. But this is a straightforward, plug 'n' chug application of equation 11.4 from your textbook, which we can rewrite as

$$P_2 - P_1 = \Delta P = \rho g h = \left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(12\ \text{m}\right) = 1.2\times 10^5\ \text{Pa}$$

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(36) In this problem, we see an application of Pascal's principle. As the brake pedal is mashed down, it pushes a piston against brake fluid which is fully enclosed by a hydraulic system. The force exerted by that piston in the "master cylinder" causes an increase in pressure, and Pascal's principle tells us that this change in pressure is immediately transmitted to every location in the fluid, without any diminution. Acting on a larger piston, this pressure increase produces a commensurately larger force which presses a brake pad against a rotor.

In your textbook, the excellent artwork accompanying this problem shows that the force applied to the pedal has a lever arm of 0.150 m with respect to a hinged arm, which pushes on the master cylinder piston at a location on the arm which is only 0.050 m from the hinge. Therefore, the force exerted on the master-cylinder piston is three times as large as the force exerted on the pedal by the driver's foot: $F_{\text{piston}} = 3\ F_{\text{pedal}}$. The change in pressure due to the pedal force, F_{pedal}, on the master-cylinder piston is equal to three times the pedal force divided by the piston area. That change in pressure exerts a force on the caliper piston equal to the pressure times the caliper piston area. Equating these:

$$\frac{\frac{0.150m}{0.050m}F_{pedal}}{\pi {r_M}^2} = \Delta P = \frac{F_{caliper}}{\pi {r_C}^2}$$

Solving for F_{caliper}:

$$F_{caliper} = 3F_{pedal}\frac{{r_C}^2}{{r_M}^2} = 3\left(9.00\ \text{N}\right)\frac{\left(1.90\times 10^{-2}\ \text{m}\right)^2}{\left(9.50\times 10^{-3}\ \text{m}\right)^2} = 108.0\ \text{N}$$

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(45) Archimedes' principle says that a fluid in which an object is immersed (in part or wholly) exerts a buoyant force on the object, whose magnitude is the weight of the fluid displaced by the object. In this problem, a cubical box floats in water, with one-third of its height submerged. We are told that the box's wall thickness is negligible, which is the authors' way of allowing us to simplify our lives by treating the interior volume and exterior volume of the box as being equal. The box has an edge length S = 0.30 m.

When it initially floats in the water, with a dry interior, a depth of (1/3)*S* is submerged. Therefore, the volume of the displaced water is $S \times S \times (1/3)S = (1/3)S^3$. If we call the mass of the dry box *m*, then as it floats in equilibrium, we can write

$$\sum F_y = \frac{1}{3}S^3\rho_W g - mg = 0$$

where ρ_W is the mass density of water (=1000 kg/m^3). We can solve this equation to find the mass of the dry box:

$$m = \frac{1}{3}\rho_W S^3$$

We now add water to the box until it is just at the point of sinking. At that point, the top of the box is exactly even with the water surface, and the volume of water displaced by the box is its entire volume of S^3. We have added water to the box to a height h, making the volume of the added water S^2h, and the added mass is $\rho_W S^2 h$. The equilibrium equation as the box is at the very edge of sinking is

$$\sum F_y = S^3\rho_W g - \frac{1}{3}\rho_W S^3 g - \rho_W S^2 hg = 0$$

(The first term is the buoyant force on the fully-immersed box; the second term is the weight of the dry box, from above; and the third term is the weight of the added water inside the box.) We solve this equation for *h*:

$$\frac{2}{3}S^3 = S^2 h$$

$$h = \frac{2}{3}S = \frac{2(0.30\text{ m})}{3} = 0.20\text{ m}$$

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(51) In this simple problem, we are asked to calculate the mass flow rate resulting from plumbing a modest volume of a solution into a person over an extended period of time. In telling us to calculate "the" mass flow rate, the problem implies without saying so that the flow is constant over the 6-hour time period; the problem might better have demanded the "average" mass flow rate, but – no matter. We are given the solution's mass density (r = 1030 kg/m^3), the dispensed volume (V = 9.5 x 10^{-4} m^3), and the time period (t = 6 hr). The mass flow rate is equal to the volume flow rate times the mass density:

$$\frac{\Delta m}{\Delta t} = \rho\frac{\Delta V}{\Delta t} = \frac{\left(1030\text{ kg/m}^3\right)\left(9.5\times10^{-4}\text{ m}^3\right)}{\left(6\text{ hr}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)} = 4.5\times10^{-5}\text{ kg/s}$$

(55) For a noncompressible fluid like water, the equation of continuity tells us that *v A*, the product of cross-sectional area and flow velocity, is constant at every point in an enclosed flow. In this problem, water flows through a pipe with an internal radius $r_0 = 6.5 \times 10^{-3}$ m with a velocity $v_0 = 1.2$ m/s toward a shower head with 12 holes, each of which has a radius $r_f = 4.6 \times 10^{-4}$ m.

To solve the problem, we must assume that the volume flow will divide itself equally into 12 identical streams – one for each of the holes in the shower head. Then, we can write the equation of continuity in the form

$$v_0 A_0 = 12 v_f A_f$$

Substituting for A_0 and A_f, and solving for v_f,

$$v_0 \cdot \pi r_0^{\,2} = 12 v_f \cdot \pi r_f^{\,2}$$

$$v_f = v_0 \frac{r_0^{\,2}}{12 r_f^{\,2}} = (1.2\ \text{m/s}) \frac{(6.5\times10^{-3}\ \text{m})^2}{12(4.6\times10^{-4}\ \text{m})^2} = 20\ \text{m/s}$$

Actually, this was the part (b) answer. Part (a) asked for the volume flow rate in the line. The volume flow rate is simply the invariant quantity *v A* in the incompressible flow.

$$A\,v = \pi\,(6.5 \times 10^{-3}\ \text{m})^2\,(1.2\ \text{m/s}) = 1.6 \times 10^{-4}\ \text{m}^3/\text{s}$$

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(59) We will use Bernoulli's equation to calculate the lifting force generated by the wing in this problem. We are told that the overwing velocity $v_o = 251$ m/s, and that the underwing velocity $v_u = 225$ m/s, in air whose mass density ρ is 1.29 kg/m^3, and that the wing has an area $A = 24.0$ m^2. The lifting force is given by

$$F_{lift} = A\,\Delta P = A\,(P_u - P_o)$$

Bernoulli's equation will give us ΔP:

$$P_o + \frac{1}{2}\rho v_o^{\,2} = P_u + \frac{1}{2}\rho v_u^{\,2}$$

(since the height *y* is essentially the same for both wing surfaces). Solve for $P_u - P_o$:

$$P_u - P_o = \frac{1}{2}\left(v_o^{\,2} - v_u^{\,2}\right)$$

Substitute this result back into the first equation:

$$F_{lift} = \frac{1}{2} A\rho\left(v_o^{\,2} - v_u^{\,2}\right) = \frac{1}{2}\left(24.0\ \text{m}^2\right)\left(1.29\ \text{kg/m}^3\right)\left[(251\ \text{m/s})^2 - (225\ \text{m/s})^2\right] = 1.92\times10^5\ \text{N}$$

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(67) This is essentially the same problem as the previous one, with different numbers and stated slightly differently. We're told that the airplane is in level flight, and we're then asked for the weight of the airplane. If the airplane is in level flight, it cannot be accelerating vertically, and so the sum of the vertical forces acting on it is zero. The vertical forces are the lift force generated by the wing and the weight force. So, what we're really being asked for is the lift force. We can calculate it by the same equation we developed in the last problem. This time, v_u = 54.0 m/s, v_o = 62.0 m/s, A = 16 m^2, and ρ is still 1.29 kg/m^3. Substituting these values, we obtain:

F_{lift} (= weight) = 9.6 x 10^3 N.

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(69) This problem is a sort of puzzle. We are told that a hinged plate is held horizontal by air being passed over its upper surface with a velocity of 11.0 m/s; then we are asked to say what velocity would be necessary if we are willing to allow the plate to "droop" to an angle of 30.0° with the vertical. The trouble is that we know neither the plate's mass, nor its area. So we will press on with the childlike confidence that things will work out, somehow.

From the first situation, with the plate horizontal: let us say that the plate has an area *A*, and a mass *m* (both unknown). We'll use the subscript "1" to refer to the underside of the plate, and "2" to refer to the upper surface. Then, Bernoulli's equation gives

$$P_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \frac{1}{2}\rho {v_2}^2$$

Since $v_1 = 0$: $P_1 - P_2 = \frac{1}{2}\rho {v_2}^2$.

Supposing that the plate is uniform, its center of gravity lies at its geometric center, and we can say that both the plate's weight force and the aerodynamic force $(P_1 - P_2)\,A$ will act at the center – the same distance *x* from the hinge. Therefore, if the plate is in equilibrium, the torques due to these forces must be equal, and thus the forces must also be equal:

$$(P_1 - P_2)Ax = mgx$$

which means that

$$\frac{mg}{A} = P_1 - P_2 = \frac{1}{2}\rho {v_2}^2 \qquad (1)$$

We will set that result to one side until we need it. Meanwhile, in the second situation, in which the plate is in equilibrium hanging 30.0° from vertical with an air velocity v_3 across its upper surface, we resolve its weight force into two components: mg sin θ, which is perpendicular to the plate, and mg cos θ, which is parallel to it. The parallel component has no lever arm and therefore produces no torque; the perpendicular component opposes the aerodynamic force at the new airflow velocity over the top surface of the plate. Again, since the plate is in equilibrium in the second situation, we can write

$$\frac{1}{2}A\rho v_3^{\ 2} = mg\sin\theta.\ \text{Solving for } v_3^2,\quad v_3^{\ 2} = \frac{mg}{A}\cdot\frac{2\sin\theta}{\rho}\ .$$

Substituting from equation (1), $v_3^{\ 2} = \frac{1}{2}\rho v_2^{\ 2}\cdot\frac{2\sin\theta}{\rho}$ which simplifies to:

$$v_3 = v_2\sqrt{\sin\theta} = (11.0\text{ m/s})\sqrt{\sin(30.0°)} = 7.78\text{ m/s}$$

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(80) The gauge pressure of the solution must be equal to the gauge pressure of the blood, if the condition described by the problem is to be true:

$$GP = \rho gh = (1030\text{ kg/m}^3)\ (9.80\text{ m/s}^2)\ (0.610\text{ m}) = 6157\text{ Pa} = \rho_{HG}\, g\, h_{HG}$$

$$h_{HG} = \frac{6157\text{ Pa}}{\rho_{\text{HG}}\, g} = \frac{6157\text{ Pa}}{(13\,600\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.0462\text{ m} = 46.2\text{ mm}$$
