1. Consider the following linear system

\[
\begin{align*}
    x + y + z + aw &= 4 \\
    2x + 3y + 4z + bw &= 16 \\
    \quad -2x + 3z + cw &= 11
\end{align*}
\]

where \((a, b, c)\) are any three consecutive digits from your School ID.

a) Write the augmented matrix for this system.

b) Find the REF and the RREF using Gaussian and Gauss-Jordan Elimination.

c) Find the solution set of the system.

d) Is it possible to find values of \(a, b, \) and \(c\) so that the system has exactly one solution? How about no solutions?

*Note:* You may use *Maple* to perform the elementary row operations. The *gausselim* or *gaussjord* commands can be used only to verify the results for REF and RREF. If *Maple* is used, submit printout of your worksheet.

2. Write down all possible RREF’s for the \(3 \times 4\) matrices. If an entry can be any number write ‘\(\_\)’.

3. Consider the matrices

\[
A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 5 \\ 1 & 6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 5 & 0 \end{bmatrix}
\]

Compute

(a) \(BC\); \quad (b) \((AC + 3B^T)^T\); \quad (c) \(tr((1/3)BAC)\).

4. A square matrix \(A\) is called **symmetric** if \(A^T = A\), and **skew-symmetric** if \(A^T = -A\).

Show that if \(B\) is a square matrix, then

(a) \(BB^T\) and \(B + B^T\) are symmetric; \quad (b) \(B - B^T\) is skew-symmetric.