Show all work! Maple can be used for verification of answers and output should be submitted with the homework (check with instructor, if necessary).

1. Let
   \[ A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 1 & 3 & -1 \\ -2 & -5 & 1 \\ 1 & 5 & -2 \end{bmatrix} \]
   (a) Solve the following systems of equations without finding \( A \)
   \[
   \begin{align*}
   ax + by + cz &= 1 \\
   (i) \quad dx + ey + fz &= 1 \\
   (ii) \quad gx + hy + iz &= 1 \\
   
   \end{align*}
   \]
   (b) Find \( A \).

2. Show that
   \[ A = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix} \]
   is not invertible for any values of the entries.

3. Indicate whether the statement is always true or sometimes false. Justify your answer with a logical argument or a counterexample.
   (a) Every square matrix can be expressed as a product of elementary matrices.
   (b) If \( A \) and \( B \) are \( n \times n \) matrices and \( AB \) is invertible, then so are \( A \) and \( B \).
   (c) If \( A \) is invertible, then \( Ax = x \) has unique solution.

4. Let \( Ax = b \) be a consistent system of linear equations, and let \( x_1 \) be a fixed solution. Show that every solution to the system can be written in the form \( x = x_1 + x_0 \), where \( x_0 \) is a solution to \( Ax = 0 \). Show also that every matrix of this form is a solution.

5. A square matrix \( A \) is called skew-symmetric if \( A^T = -A \). Prove:
   (a) If \( A \) is an invertible skew-symmetric matrix, then \( A^{-1} \) is skew-symmetric.
   (b) If \( A \) and \( B \) are skew-symmetric, then so are \( A^T \), \( A + B \), \( A - B \), and \( kA \) for any scalar \( k \).