Show all work! Maple can be used for verification of answers and output should be submitted with the homework (check with instructor, if necessary).

1. Let \( \mathbf{u} = (1, 1, -4) \), \( \mathbf{v} = (1, 1, 1) \), and \( \mathbf{w} = (1, -4, 1) \).
   
   (a) Find the components of \( \mathbf{u} - \mathbf{v} \), \( 6\mathbf{u} + 2\mathbf{v} \), and \( 2(3\mathbf{u} - \mathbf{v}) + (5\mathbf{v} - 7\mathbf{w}) \).
   
   (b) Find the components of a vector \( \mathbf{x} \), such that \( 2\mathbf{u} - \mathbf{v} + \mathbf{x} = 7\mathbf{x} + \mathbf{w} \).
   
   (c) Find scalars \( c_1 \), \( c_2 \), and \( c_3 \), such that
   \[
   c_1 \mathbf{u} + c_2 \mathbf{v} + c_3 \mathbf{w} = (5, 10, 0).
   \]

2. (a) Prove geometrically that if \( \mathbf{u} \) and \( \mathbf{v} \) are vectors in 2- or 3- space, then \( \|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\| \).

   (b) Prove analytically the same inequality for 2- space.

   (c) Is it possible to have \( \|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\| \)? Explain your reasoning.

3. (a) Show that if \( \mathbf{v} \) is orthogonal to both \( \mathbf{w}_1 \) and \( \mathbf{w}_2 \), then \( \mathbf{v} \) is orthogonal to \( k_1 \mathbf{w}_1 + k_2 \mathbf{w}_2 \) for all scalars \( k_1 \) and \( k_2 \).

   (b) Let \( \mathbf{u} \) and \( \mathbf{v} \) be nonzero vectors in 2- or 3- space, and let \( k = \|\mathbf{u}\| \) and \( l = \|\mathbf{v}\| \). Show that the vector \( \mathbf{w} = \frac{k}{l} \mathbf{u} + k \mathbf{v} \) bisects the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

4. Consider the parallelepiped with sides \( \mathbf{u} = (3, 2, 1) \), \( \mathbf{v} = (1, 1, 2) \), and \( \mathbf{w} = (1, 3, 3) \).

   (a) Find the volume of this parallelepiped.

   (b) Find the area of the face determined by \( \mathbf{u} \) and \( \mathbf{w} \).

   (c) Find the angle between \( \mathbf{u} \) and the plane containing the face determined by \( \mathbf{v} \) and \( \mathbf{w} \).

   [Note: The angle between a vector and a plane is defined to be the complement of the angle \( \theta \) between the vector and that normal to the plane for which \( 0 \leq \theta \leq \pi/2 \).]