Que 1. Indicate which of the following is an all-integer program and which is a mixed-integer linear …

a. This is a mixed integer linear program. Its LP Relaxation is

\[
\begin{align*}
\text{Max} & \quad 30x_1 + 25x_2 \\
\text{s.t.} & \quad 3x_1 + 1.5x_2 \leq 400 \\
& \quad 1.5x_1 + 2x_2 \leq 250 \\
& \quad x_1 + x_2 \leq 150 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

b. This is an all-integer linear program. Its LP Relaxation just requires dropping the words "and integer" from the last line.

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Que 3. Consider the following all-integer linear program.

a.

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b. The optimal solution to the LP Relaxation is shown on the above graph to be \(x_1 = 4, x_2 = 1\). Its value is 5.

c. The optimal integer solution is the same as the optimal solution to the LP Relaxation. This is always the case whenever all the variables take on integer values in the optimal solution to the LP Relaxation.
Que 5 Consider the following mixed-integer linear program.

a.

The feasible mixed integer solutions are indicated by the boldface vertical lines in the graph above.

b. The optimal solution to the LP relaxation is given by $x_1 = 3.14, x_2 = 2.60$. Its value is 14.08. Rounding the value of $x_1$ down to find a feasible mixed integer solution yields $x_1 = 3, x_2 = 2.60$ with a value of 13.8. This solution is clearly not optimal. With $x_1 = 3$ we can see from the graph that $x_2$ can be made larger without violating the constraints.

c.

The optimal solution to the MILP is given by $x_1 = 3, x_2 = 2.67$. Its value is 14.
Que 7. The following questions refer to a capital budgeting problem with six projects represented by 0, 1
a. \( x_1 + x_3 + x_5 + x_6 = 2 \)
b. \( x_3 - x_5 = 0 \)
c. \( x_1 + x_4 = 1 \)
d. \( x_4 \leq x_1 \),
\( x_4 \leq x_3 \)
e. \( x_4 \leq x_1 \),
\( x_4 \leq x_3 \),
\( x_4 \geq x_1 + x_3 - 1 \)

Que 9. Hawkins Manufacturing Company produces connecting rods for 4- and 6-cylinder automobile …
a. \( x_4 \leq 8000 \) s
b. \( x_6 \leq 6000 \) s
c. \( x_4 \leq 8000 \) s,
\( x_6 \leq 6000 \) s,
\( s_4 + s_6 = 1 \)
d. Min \( 15 x_4 + 18 x_6 + 2000 s_4 + 3500 s_6 \)

Que. 11 Hart Manufacturing makes three products. Each product requires manufacturing operations in …
a. Let \( P_i \) = units of product \( i \) produced

Max \[ \begin{array}{c}
25P_1 + 28P_2 + 30P_3 \\
\end{array} \]
s.t. \[ \begin{array}{c}
1.5P_1 + 3P_2 + 2P_3 \leq 450 \\
2P_1 + 1P_2 + 2.5P_3 \leq 350 \\
0.25P_1 + 0.25P_2 + 0.25P_3 \leq 50 \\
P_1, P_2, P_3 \geq 0
\end{array} \]
b. The optimal solution is
   \[ P_1 = 60 \]
   \[ P_2 = 80 \quad \text{Value} = 5540 \]
   \[ P_3 = 60 \]
This solution provides a profit of $5540.

c. Since the solution in part (b) calls for producing all three products, the total setup cost is
   \[ $1550 = $400 + $550 + $600. \]
Subtracting the total setup cost from the profit in part (b), we see that
   \[ \text{Profit} = $5540 - 1550 = $3990 \]

d. Introduce a 0-1 variable \( y_i \) that is 1 if any quantity of product \( i \) is produced and 0 otherwise.

   With the maximum production quantities provided by management, we get 3 new constraints:
   \[ P_1 \leq 175y_1 \]
   \[ P_2 \leq 150y_2 \]
   \[ P_3 \leq 140y_3 \]

   Bringing the variables to the left-hand side of the constraints, we obtain the following fixed charge formulation of the Hart problem.
   \[
   \begin{align*}
   \text{Max} & \quad 25P_1 + 28P_2 + 30P_3 - 400y_1 - \frac{550y_2}{2} - 600y_3 \\
   \text{s.t.} & \quad 1.5P_1 + 3P_2 + 2P_3 \leq 450 \\
   & \quad 2P_1 + P_2 + 2.5P_3 \leq 350 \\
   & \quad .25P_1 + .25P_2 + .25P_3 \leq 50 \\
   & \quad P_1 - 175y_1 \leq 0 \\
   & \quad P_2 - 150y_2 \leq 0 \\
   & \quad P_3 - 140y_3 \leq 0 \\
   & P_1, P_2, P_3 \geq 0; \ y_1, y_2, y_3 = 0, 1
   \end{align*}
   \]

e. The optimal solution using The Management Scientist is
   \[ P_1 = 100 \quad y_1 = 1 \]
   \[ P_2 = 100 \quad y_2 = 1 \quad \text{Value} = 4350 \]
   \[ P_3 = 0 \quad y_3 = 0 \]

   The profit associated with this solution is $4350. This is an improvement of $360 over the solution in part (c).
Que 15. CHB, Inc., is a bank holding company that is evaluating the potential for expanding into a 13- 

a. Let \( x_i = \begin{cases} 
1 & \text{if a principal place of business in county } i \\
0 & \text{otherwise} 
\end{cases} \)

\( y_i = \begin{cases} 
1 & \text{if county } i \text{ is not served} \\
0 & \text{if county } i \text{ is served} 
\end{cases} \)

The objective function for an integer programming model calls for minimizing the population not served.

\[
\text{Min } 195y_1 + 96y_2 + \cdots + 175y_{13}
\]

There are 13 constraints needed; each is written so that \( y_i \) will be forced to equal one whenever it is not possible to do business in county \( i \).

\[
\text{Constraint 1: } x_1 + x_2 + x_3 + y_1 \geq 1 \\
\text{Constraint 2: } x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + y_2 \geq 1 \\
\vdots \\
\text{Constraint 13: } x_{11} + x_{12} + x_{13} + y_{13} \geq 1 
\]

One more constraint must be added to reflect the requirement that only one principal place of business may be established.

\[
x_1 + x_2 + \cdots + x_{13} = 1
\]

The optimal solution has a principal place of business in County 11 with an optimal \((z)\) value of 739,000. A population of 739,000 cannot be served by this solution. Counties 1-5 and 10 will not be served. This implies that counties 6, 7, 8, 9, 11, 12 and 13 with a total population of 961,000 will be served.

b. The only change necessary in the integer programming model for part a is that the right-hand side of the last constraint is increased from 1 to 2.

\[
x_1 + x_2 + \cdots + x_{13} = 2
\]

The optimal solution has principal places of business in counties 3 and 11 with an optimal value of 76,000. Only County 10 with a population of 76,000 is not served – implying that the rest (a population of 1,624,000) is served.

c. County 5 is not the best location if only one principal place of business can be established; only 642,000 can be served – that means 1,058,000 customers in the region cannot be served. However, and if there is no opportunity to obtain a principal place of business in County 11, County 5 may be a good start. Perhaps later there will be an opportunity in County 11.
Que 16. The Northshore bank is working to develop an efficient work schedule for full-time and part- ... 

a. 

\[
\begin{align*}
\min & \quad 105x_9 + 105x_{11} + 105x_{10} + 32y_9 + 32y_{10} + 32y_{11} + 32y_{12} + 32y_1 + 32y_2 + 32y_3 \\
\text{s.t.} & \quad x_9 + x_{10} + y_9 + y_{10} \geq 6 \\
& \quad x_9 + x_{10} + x_{11} + y_9 + y_{10} + y_{11} \geq 8 \\
& \quad x_9 + x_{10} + x_{11} + y_9 + y_{10} + y_{11} + y_{12} \geq 10 \\
& \quad x_{10} + x_{11} + y_{10} + y_{11} + y_{12} + y_1 \geq 9 \\
& \quad x_9 + x_{10} + x_{11} + y_{11} + y_{12} + y_1 + y_2 \geq 6 \\
& \quad x_9 + x_{10} + x_{11} + y_{12} + y_1 + y_2 + y_3 \geq 4 \\
& \quad x_9 + x_{10} + x_{11} + y_2 + y_3 \geq 7 \\
& \quad x_{10} + x_{11} + y_3 \geq 6 \\
& \quad x_i, \ y_j \geq 0 \text{ and integer for } i = 9, 10, 11 \text{ and } j = 9, 10, 11, 12, 1, 2, 3
\end{align*}
\]

b. Solution to LP Relaxation obtained using LINDO/PC:

\[
\begin{align*}
y_9 &= 6 \\
y_{12} &= 6 \\
y_3 &= 6 \\
y_{11} &= 2 \\
y_1 &= 1 \\
\text{Cost: } & \quad \text{ $672.}$
\end{align*}
\]

c. The solution to the LP Relaxation is integral therefore it is the optimal solution to the integer program.

A difficulty with this solution is that only part-time employees are used; this may cause problems with supervision, etc. The large surpluses from 5, 12-1 (4 employees), and 3-4 (9 employees) indicate times when the tellers are not needed for customer service and may be reassigned to other tasks.

d. Add the following constraints to the formulation in part (a).

\[
\begin{align*}
x_9 & \geq 1 \\
x_{11} & \geq 1 \\
x_9 + x_{10} + x_{11} & \geq 5
\end{align*}
\]

The new optimal solution, which has a daily cost of $909 is

\[
\begin{align*}
x_9 &= 1 \\
x_{11} &= 4 \\
y_9 &= 5 \\
y_{12} &= 5 \\
y_3 &= 2
\end{align*}
\]

There is now much less reliance on part-time employees. The new solution uses 5 full-time employees and 12 part-time employees; the previous solution used no full-time employees and 21 part-time employees.
Que 25. East Coast trucking provides service from Boston to Miami using regional offices located …

a. Let \( x_i = \begin{cases} 
1 & \text{if a service facility is located in city } i \\
0 & \text{otherwise} 
\end{cases} \)

\[
\begin{align*}
\min & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \\
\text{s.t.} & \quad x_1 + x_2 + x_3 \geq 1 \\
& \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \\
& \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 1 \\
& \quad x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 1 \\
& \quad x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1 \\
& \quad x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \geq 1 \\
& \quad x_6 + x_7 + x_8 + x_9 + x_{10} \geq 1 \\
& \quad x_7 + x_8 + x_9 + x_{10} + x_{11} \geq 1 \\
& \quad x_8 + x_9 + x_{10} + x_{11} \geq 1 \\
& \quad x_9 + x_{10} + x_{11} + x_{12} \geq 1 \\
& \quad x_{11} + x_{12} \geq 1 \\
& \quad x_i = 0, 1
\end{align*}
\]

b. 3 service facilities: Philadelphia, Savannah and Tampa.

Note: alternate optimal solution is New York, Richmond and Tampa.

c. 4 service facilities: New York, Baltimore, Savannah and Tampa.

Note: alternate optimal solution: Boston, Philadelphia, Florence and Tampa.