(9) The volume of a rectangular solid is given by the product of its width, length, and depth: 
\[ V = \Delta x \cdot \Delta y \cdot \Delta z \]. Since we desire a volume expressed in the SI units of m³, we must convert each dimension to meters.

\[ \Delta x = (1.20 \text{ naut mi}) \left( \frac{6076 \text{ ft}}{1 \text{ naut mi}} \right) \left( \frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 2222.358 \text{ m} \]

\[ \Delta y = (2.60 \text{ naut mi}) \left( \frac{6076 \text{ ft}}{1 \text{ naut mi}} \right) \left( \frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 4815.108 \text{ m} \]

\[ \Delta z = (16.0 \text{ fathoms}) \left( \frac{6 \text{ ft}}{1 \text{ fathom}} \right) \left( \frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 29.26080 \text{ m} \]

\[ V = \Delta x \cdot \Delta y \cdot \Delta z = (2222.358 \text{ m})(4815.108 \text{ m})(29.26080 \text{ m}) = 3.13 \times 10^8 \text{ m}^3 \]

Note: in general, you are better off solving the entire problem algebraically, writing down a single expression for the desired answer before substituting any numerical values. However, if you choose to calculate intermediate numerical results, as we did here, carry some extra significant figures in order to avoid rounding errors. When you calculate your final numerical result, express it with the appropriate number of significant figures – that is, the smallest number of significant figures that is found in the problem data.

(10) To determine the necessary dimensions of the spring constant \( k \), we will solve the formula for \( T \), given in the problem, for \( k \); then we will substitute the dimensions of the variables on the right-hand side and simplify.

\[ T = 2\pi \sqrt{\frac{m}{k}} \quad T^2 = 4\pi^2 \frac{m}{k} \quad T^2 k = 4\pi^2 m \quad k = \frac{4\pi^2 m}{T^2} \]

\( 4\pi^2 \) is a dimensionless numerical constant, so the dimensions of \( k \) must be the dimensions of \( \frac{m}{T^2} : \text{ mass}/ \text{time}^2 \).
(12) Drawing a picture is often a productive part of your problem-solving strategy:

\[ L = 2830 \text{ m} \]

\[ \theta = 14.6^\circ \]

From the right triangle, we see that the sine of the given angle is equal to the length of the side opposite that angle divided by the length of the hypotenuse:

\[ \sin \theta = \frac{y}{L} \]

Solving for \( y \):

\[ y = L \sin \theta = (2830 \text{ m})(\sin 14.6^\circ) = 713 \text{ m} \]

(16) There are at least two good ways to solve this problem, and we’ll do both of them here.

The picture above shows us that \( AC \), the distance for which we’re asked, is the hypotenuse of a right triangle \( ABC \). One leg, \( BC \), has the cube side length \( s \); the other leg, \( AB \), is the hypotenuse of another right triangle, which has two legs of equal length \( s \). We can use the Pythagorean theorem to write
\[ AC^2 = s^2 + (s^2 + s^2) = 3s^2 \]

\[ AC = \sqrt{3s^2} = s\sqrt{3} = 0.281 \text{ nm} \cdot \sqrt{3} = 0.487 \text{ nm} \]

Alternatively, we could simply apply the distance formula. Choosing one corner of the cube as the origin (0, 0, 0), the opposite corner has coordinates (s, s, s). The distance between these opposite corners is

\[ D = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} \]

\[ D = \sqrt{(s - 0)^2 + (s - 0)^2 + (s - 0)^2} = \sqrt{3s^2} = \sqrt{3} (0.281 \text{ nm}) = 0.487 \text{ nm} \]

(25) This problem provides an example of the simplest case of adding vectors: when two vectors are parallel (or antiparallel). The vector \( \mathbf{A} \) is a northward displacement of 2.43 km, and \( \mathbf{B} \) is a northward displacement of 7.74 km. Part (a) asks us for the magnitude and direction of the vector \( (\mathbf{A} - \mathbf{B}) \). This is simply \( (2.43 \text{ km} - 7.74 \text{ km}) \), northward. Of course, the magnitude is also less than zero: \( 2.43 \text{ km} - 7.74 \text{ km} = -5.31 \text{ km} \). This means that the resultant displacement vector is 5.31 km, southward.

In part (b), we must determine the vector \( (\mathbf{B} - \mathbf{A}) \). The magnitude is \( 7.74 \text{ km} - 2.43 \text{ km} = 5.31 \text{ km} \); the direction is northward.

(39) The drawing in your textbook shows us that the 475-N force vector makes an angle of 54.0° with the +Z axis. We can, then, express the force vector as the sum of two vectors: one, \( F_Z \), in the +Z direction, and the other, \( F_{XY} \), in the X-Y plane.

Simple right-triangle trigonometry gives us

\[ F_Z = F \cos(54.0^\circ) = (475 \text{ N}) \cdot \cos(54.0^\circ) = 279 \text{ N} \]

\[ F_{XY} = F \sin(54.0^\circ) \]

In the X-Y plane, we can use the 33.0° angle that \( F_{XY} \) makes with the X-axis to decompose \( F_{XY} \) into X and Y components, \( F_X \) and \( F_Y \):
\[ F_x = F_{xy} \cos(33.0^\circ) = F \sin(54.0^\circ) \cos(33.0^\circ) = 475 \text{ N} \cdot \sin(54.0^\circ) \cdot \cos(33.0^\circ) = 322 \text{ N} \]

\[ F_y = F_{xy} \sin(33.0^\circ) = F \sin(54.0^\circ) \sin(33.0^\circ) = 475 \text{ N} \cdot \sin(54.0^\circ) \cdot \sin(33.0^\circ) = 209 \text{ N} \]

(50) In solving this problem, we will call displacements toward the east and north positive, and displacements toward the west and south negative. Our strategy will be to resolve each of the grasshopper’s four displacements into north-south and east-west components. Then we will add up the components, and use the component sums to determine the total displacement's magnitude and direction. We will organize these calculations into convenient table form.

<table>
<thead>
<tr>
<th>displacement #</th>
<th>east component calculation</th>
<th>north component calculation</th>
<th>east component value</th>
<th>north component value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- 27.0 cm</td>
<td>0 cm</td>
<td>- 27.0 cm</td>
<td>0 cm</td>
</tr>
<tr>
<td>2</td>
<td>(-23.0 cm) \cdot \cos 35.0^\circ</td>
<td>(-23.0 cm) \cdot \sin 35.0^\circ</td>
<td>-18.84 cm</td>
<td>-13.19 cm</td>
</tr>
<tr>
<td>3</td>
<td>(28.0 cm) \cdot \cos 55.0^\circ</td>
<td>-(28.0 cm) \cdot \sin 55.0^\circ</td>
<td>16.06 cm</td>
<td>-22.94 cm</td>
</tr>
<tr>
<td>4</td>
<td>(35.0 cm) \cdot \cos 63.0^\circ</td>
<td>(35.0 cm) \cdot \sin 63.0^\circ</td>
<td>15.89 cm</td>
<td>31.19 cm</td>
</tr>
</tbody>
</table>

TOTALS: -13.89 cm -4.94 cm

Since we have negative values for the total east and north displacement components, we know that we actually have displacement components toward the south and the west. The magnitude of the total displacement is

\[ \text{Displacement} = \sqrt{(-13.89 \text{ cm})^2 + (-4.94 \text{ cm})^2} = 14.7 \text{ cm} \]

The direction, relative to due west, is given by

\[ \theta = \arctan\left(\frac{-4.94 \text{ cm}}{-13.89 \text{ cm}}\right) = 19.6^\circ \]

So, the total or resultant displacement is 14.7 cm, 19.6° south of west.
Here, we must add two displacement vectors to determine the bear’s total displacement, then prescribe a displacement vector that will return the bear from his final position back to his starting position. The return vector will be simply the total outward-travel displacement vector multiplied by the scalar -1; that means that the return vector will be antiparallel to the original total displacement vector.

A decent picture is usually quite indispensable in these problems:

Our strategy here is to resolve both vectors into their X and Y components, add up those components, and then calculate the magnitude and direction of the resulting vector. Here, we have chosen our coordinate system so that one of the two vectors is parallel to a coordinate axis – in this case, the X axis, which we choose to be directed east. Now the magnitudes of the components are:

\[ A_X = -1563 \text{ m} \quad A_Y = 0 \]
\[ B_X = -(3348 \text{ m}) \cos 32.0^\circ = -2839.27 \text{ m} \quad B_Y = (3348 \text{ m}) \sin 32.0^\circ = 1774.17 \text{ m} \]

Adding the component magnitudes:

\[ C_X = (-1563 \text{ m}) + (-2839.27 \text{ m}) = -4402.27 \text{ m} \]
\[ C_Y = (0 \text{ m}) + (1774.17 \text{ m}) = 1774.17 \text{ m} \]

The total magnitude:

\[ C = \sqrt{C_X^2 + C_Y^2} = \sqrt{(-4402.27 \text{ m})^2 + (1774.17 \text{ m})^2} = 4746.33 \text{ m} \]

The direction:

\[ \theta = \arcsin \left( \frac{C_Y}{C} \right) = \arcsin \left( \frac{1774.17 \text{ m}}{4746.33 \text{ m}} \right) = 22.0^\circ \text{ (north of west)} \]

The return displacement vector: 4746 m, 22.0° south of east.