(5) The bullet accelerates from rest \((v_0 = 0)\) to \(v = 715\, \text{m/s}\) in a time \(t = 2.50 \times 10^{-3}\, \text{s}\). Its average acceleration is

\[
a = \frac{v - v_0}{t} = \frac{715\, \text{m/s} - 0\, \text{m/s}}{2.50 \times 10^{-3}\, \text{s}} = 2.86 \times 10^5\, \text{m/s}^2.
\]

Newton’s second law gives us the average net force on the bullet during its acceleration:

\[
F = ma = \left(1.5 \times 10^{-2}\, \text{kg}\right) \left(2.86 \times 10^5\, \text{m/s}^2\right) = 4290\, \text{N}
\]

(15) A picture is usually helpful:

And yes, I know that’s a bunny in the picture, not a duck as the problem calls out. However, as it turns out, I didn’t have any clip art of a duck handy … whereas the bunny was readily available. So it goes.

Can we get the duck’s displacement from the forces applied to him? Not directly. Our strategy will be to use our knowledge of dynamics to get acceleration from force and mass, and then to use our kinematics to get displacement from acceleration and time. We will use the powerful idea that the accelerations may be calculated and applied in two orthogonal directions independently, and combined at the end.

The force \(F\) due to the duck’s paddling acts parallel to one of the coordinate axes (east), so we don’t need to express it as a sum of components. The same is not true of the force due to the current, \(F_C\). Therefore, we will resolve it into east and south components:

\[
F_{CE} = F_C \cos \theta \quad \text{and} \quad F_{CS} = F_C \sin \theta
\]

We can calculate the acceleration in each direction by applying Newton’s second law to each direction:

\[
a_E = \frac{\sum F_E}{m} = \frac{F + F_{CE}}{m} = \frac{F + F_C \cos \theta}{m} \quad \text{and} \quad a_S = \frac{\sum F_S}{m} = \frac{F_{CS}}{m} = \frac{F_C \sin \theta}{m}
\]
where \( m \) is the duck’s mass.

In each direction, we can now calculate the displacement of the duck that will occur in a time \( t \). We will call the eastward displacement \( x \), and the southward displacement \( y \):

\[
x = v_{0e} t + \frac{1}{2} a_e t^2 = v_{0e} t + \frac{1}{2} \left( \frac{F + F_c \cos \theta}{m} \right) t^2
\]

\[
y = v_{0s} t + \frac{1}{2} a_s t^2 = v_{0s} t + \frac{1}{2} \left( \frac{F_c \sin \theta}{m} \right) t^2
\]

At this point, we note that there is no initial southward velocity, so the expression for \( y \) simplifies to \( y = \frac{1}{2} \left( \frac{F_c \sin \theta}{m} \right) t^2 \).

We’re almost finished. We now need to combine the eastward and southward displacements to obtain a total displacement (magnitude and direction).

The total magnitude: \( d = \sqrt{x^2 + y^2} \), or (substituting our earlier results):

\[
d = \sqrt{\left( v_{0e} t + \frac{1}{2} \left( \frac{F + F_c \cos \theta}{m} \right) t^2 \right)^2 + \left( \frac{1}{2} \left( \frac{F_c \sin \theta}{m} \right) t^2 \right)^2}
\]

\[
d = \sqrt{\left( 0.11 \text{ m/s} \right) \left( 3.0 \text{ s} \right) + \left[ \frac{0.10 \text{ N} + \left( 0.20 \text{ N} \right) \cos 52^\circ}{2\left( 2.5 \text{ kg} \right)} \right] \left( 3.0 \text{ s} \right)^2 + \left[ \frac{(0.20 \text{ N}) \sin 52^\circ}{2(2.5 \text{ kg})} \right] \left( 3.0 \text{ s} \right)^2}
\]

\[
d = 0.79 \text{ m}
\]

And the direction: \( \theta = \arctan \left( \frac{y}{x} \right) = \arctan \left( \frac{\frac{1}{2} \left( \frac{F_c \sin \theta}{m} \right) t^2}{v_{0e} t + \frac{1}{2} \left( \frac{F + F_c \cos \theta}{m} \right) t^2} \right) \)

\[
\theta = \arctan \left( \frac{\left[ \frac{(0.20 \text{ N}) \sin 52^\circ}{2(2.5 \text{ kg})} \right] \left( 3.0 \text{ s} \right)^2}{\left( 0.11 \text{ m/s} \right) \left( 3.0 \text{ s} \right) + \left[ \frac{0.10 \text{ N} + \left( 0.20 \text{ N} \right) \cos 52^\circ}{2\left( 2.5 \text{ kg} \right)} \right] \left( 3.0 \text{ s} \right)^2} \right) = 21^\circ, \text{ south of east.} \)
(20) Two forces act on the stone in this problem. One is its weight; the other is the air-resistance force. The drag or viscous force exerted on the stone by the air it passes through would, in real life, be a function of the stone’s velocity; but we’ll pass over that for now, and treat this force as a constant one.

Choosing our coordinate system so that +Y points upward, we write down Newton’s second law as it applies to the stone:

\[ \sum F_y = F_{\text{air}} - mg = ma_y , \]

where \( F_{\text{air}} \) is the upward-directed air resistance force. Solving this equation for \( a_y \), we have

\[ a_y = \frac{F_{\text{air}}}{m} - g = \frac{18 \text{ N}}{45 \text{ kg}} - 9.8 \text{ m/s}^2 = -9.4 \text{ m/s}^2 . \]

The problem asks us for the magnitude of the stone’s acceleration, so we lose the minus sign, which told us that the acceleration is downward. The magnitude is 9.4 m/s².

(29) This is a good problem, though semi-tricky. Let’s call the combined mass of the balloon and its original payload \( M \), and the mass to be tossed overboard \( m \). Originally, the balloon is in an equilibrium condition; that means Newton’s first law is satisfied. Written for the vertical direction (with “+” meaning “up”), that tells us that

\[ M \gg F_B = Mg \]

\( Mg \), the downward force, is simply the weight of balloon plus its original contents; and \( F_B \) is the upward buoyant force. Later, after the unwanted equipment has been discarded (or the unpopular passenger made to walk the plank), the balloon and its reduced burden have an upward acceleration, \( a \). Now it is required to satisfy Newton’s second law:

\[ \sum F = F_B - (M - m)g = (M - m)a . \]

We simply substitute for \( F_B \), according to the equation we wrote based on the earlier equilibrium condition, and solve for \( m \):

\[ F_B = (M - m)g = (M - m)a \]

\[ Mg - (M - m)g = (M - m)a \]

\[ Mg - Mg + mg = Ma - ma \]

\[ ma + mg = Ma \]

\[ m = \frac{Ma}{a + g} = \frac{(310 \text{ kg})(0.15 \text{ m/s}^2)}{0.15 \text{ m/s}^2 + 9.8 \text{ m/s}^2} = 4.67 \text{ kg} \]

or 4.7 kg, to two significant figures. (At least now we know no one walked the plank!)
In this somewhat-complex problem, we’re dealing with several applications of all three of Newton's laws of motion.

In this problem, an unknown horizontal force \( P \) pulls the large block, accelerating it toward the right. The acceleration is large enough that a small block resting against the leading face of the large one does not slide down it. We’re told that the large block is on a frictionless surface, and we’re given the masses, \( M \) and \( m \), of the large and small blocks, along with the coefficient of static friction, \( \mu_S \), between them; and then we’re asked to find the smallest force \( P \) that will do the job. Intuitively, we can see how the solution will go: the coefficient of static friction will tell us what normal force will be required to generate a static frictional force equal to the weight of the small block; given that normal force and the mass of the small block, we can calculate the acceleration that accompanies it; and given the masses of both blocks, we can calculate the magnitude of the force \( P \) required to produce that acceleration.

The two free-body diagrams above will be useful to us: one for the large block, and one for the small one. Look at the small block first. Three forces act on it: its downward weight force, its upward static frictional force, and a normal force exerted on it by the large block. Let’s call the rightward acceleration magnitude \( a \). Then, applying Newton’s second law in the horizontal direction, we have

\[
\sum F_x = N' = ma
\]

while in the vertical direction, the small block is in equilibrium:

\[
\sum F_y = F_s - mg = 0 \quad \text{Substitute for } F_s, \quad F_s = \mu_S N' : \quad \mu_S N' - mg = 0
\]

Substituting for \( N' \), using the first equation above: \( \mu_S ma - mg = 0 \). Notice, at this point, that \( m \) is going to divide out of this equation as we solve it for \( a \) … does this mean that the authors of your text were merely perpetuating a cruel hoax against us, when they provided a numerical value for \( m \) in the problem? As it turns out, \( m \) will still be useful, further along in the problem. Solving now for \( a \):

\[
a = \frac{g}{\mu_S}
\]

Having determined the acceleration that the two-block system must experience, it is a relatively straightforward task to determine the magnitude of the force, \( P \), needed to produce it. Refer to
the left-hand free-body diagram. In the horizontal direction, only one force acts on the system of
two blocks: the pulling force \( P \). Newton’s second law then requires that

\[
\sum F_x = P = (M + m)a.
\]

Substituting our previous result for \( a \), we have

\[
P = (M + m) \frac{g}{\mu_s}.
\]

Now we substitute the numerical values from the problem:

\[
P = (25 \text{ kg} + 4.0 \text{ kg}) \frac{9.8 \text{ m/s}^2}{0.71} = 400 \text{ N}.
\]

(50) We can exploit the obvious symmetries in this situation to simplify our solution. We see
that the two tension forces, \( T \) and \( T' \), have the same magnitude (21.0 N), which we will just call \( T \). Since both forces make the same angle \((\theta = 16.0^\circ)\) with the horizontal, when we resolve them
into X-Y components to add them up, their horizontal components will be equal in magnitude and opposite in direction and will add to zero; so the entire net force on the tooth will be in the
vertical or Y direction.

\[
\mathbf{T}_y = T \sin \theta = (21.0 \text{ N})(\sin 16.0^\circ) = 5.79 \text{ N}
\]

The net force on the tooth is the sum of the Y components of both tension forces, or twice \( \mathbf{T}_y \), which is 2 (5.79 N) = 11.6 N, to three significant figures.

(63) Since your textbook has a nice drawing showing the forces, we will let that drawing serve us
as a free-body diagram. Calling the forces in the X and Y directions \( F_x \) and \( F_y \), respectively, and
calling the mass of the object \( m \), we’ll apply Newton’s second law to find the X and Y
components of the resulting acceleration:

\[
a_x = \frac{F_x}{m} \quad \text{and} \quad a_y = \frac{F_y}{m}.
\]

The magnitude of the resulting acceleration is given by

\[
a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left( \frac{F_x}{m} \right)^2 + \left( \frac{F_y}{m} \right)^2} = \sqrt{\left( \frac{40.0 \text{ N}}{4.00 \text{ kg}} \right)^2 + \left( \frac{60.0 \text{ N}}{4.00 \text{ kg}} \right)^2} = 18.0 \text{ m/s}^2
\]

The direction:
Here’s a problem not involving equilibrium, but instead the lack of it. One block hangs from a cord over a pulley; the cord’s other end is attached to a second block, which is supported frictionlessly by a magic tabletop. We’re given the weights of the two blocks, and we’re asked for the acceleration of the system, as well as the tension in the cord.

First, let’s get the weight business out of the way. We’re going to need the masses of both blocks. So, we’ll start out by noting that

\[ W_1 = m_1 g \quad W_2 = m_2 g \]
\[ m_1 = \frac{W_1}{g} \quad m_2 = \frac{W_2}{g} \]

Now, we’ll draw free-body diagrams for both blocks individually:

Newton’s second law in the horizontal direction in the left-hand diagram tells us:

\[ \sum F_x = T = m_1 a \]

and, looking vertically at the right-hand diagram:

\[ \sum F_y = T - m_2 g = m_2 (-a) \]

The reason the \( a \) in the second equation acquired a minus sign is that its direction is downward (opposed to \( T \), but we consider it to have a positive magnitude – and these equations are written in terms of magnitudes. Solving both equations for \( T \), and equating the resulting expressions for \( T \), gives us

\[ m_1 a = m_2 g - m_2 a \]

Solving for \( a \),
\[ a = \frac{m_2g}{m_1 + m_2} = \frac{W_2}{W_1 + W_2} = \frac{W_2g}{g} = \frac{(185 \text{ N})(9.80 \text{ m/s}^2)}{422 \text{ N} + 185 \text{ N}} = 2.99 \text{ m/s}^2 \]

Substituting this result back into the first equation we solved for \( T \), above, gives

\[ T = m_1a = \frac{W_1}{g}a = \frac{422 \text{ N}}{9.8 \text{ m/s}^2} \cdot 2.99 \text{ m/s}^2 = 129 \text{ N} \]

---

(82) This is both a kinematics problem and a dynamics problem. We’ll solve the kinematics part first. We know that the truck has an initial velocity, \( v_0 \), of 25.0 m/s, and will then experience a constant acceleration that will decrease its velocity to zero in an unknown distance \( x \) that will depend on the magnitude of the acceleration. We can find that dependence explicitly from the kinematic equation \( v_f^2 = v_0^2 - 2ax \). (We have given the term in \( a \) a minus sign, because the acceleration will be in the opposite direction to the initial velocity, but we’re using positive numbers for the magnitudes of both.) Noting that \( v_f = 0 \) (the truck stops), we solve for \( x \):

\[ x = \frac{v_0^2}{2a} \]

We will return to this result later … after we’ve determined the maximum acceleration.

The force required to accelerate the truck results from the friction between its tires and the road surface (or possibly a parachute brake, or a retrorocket, or a Star Trek tractor beam, or who-knows-what … we don’t care). However it is that the truck slows down, the force that accelerates the crate comes from the friction between the crate and the truck bed. Specifically, since the crate does not slide forward on the truck bed, it is the force of static friction. The greater the static frictional force, the larger can be the truck’s acceleration without making the crate slip. The maximum static frictional force is

\[ F_s = \mu_sN \]

where \( \mu_s \) is the coefficient of static friction between the crate and the truck bed, and \( N \) is the magnitude of the normal force. We know that the normal force is equal in magnitude to the weight of the crate, since the crate does not accelerate upward into the air or downward through the truck bed. So, if the mass of the crate is \( m \), the normal force is \( N = mg \), and we can rewrite the frictional force as

\[ F_s = \mu_s mg \]

Newton’s second law tells us that this frictional force must also be equal to \( ma \), the crate’s mass times its acceleration:

\[ F_s = \mu_s mg = ma \]

Solving for \( a \):

\[ a = \mu_s g \]

If we substitute this result into our kinematic result from the beginning of our work, we obtain

\[ x = \frac{v_0^2}{2a} = \frac{v_0^2}{2\mu_s g} = \frac{(25.0 \text{ m/s})^2}{2 \cdot 0.650 \cdot 9.80 \text{ m/s}^2} = 49.1 \text{ m} \]
The force applied to the ball results in a constant acceleration, given the ball’s mass. We do not know the acceleration, but we do know the initial velocity (zero), the final velocity, and the distance over which the acceleration occurs. Since the acceleration is constant, we are entitled to use the kinematic equations, such as

\[ v_f^2 = v_0^2 + 2aS \]

We can solve this equation for \( a \), and then substitute our result into Newton’s second law to determine the force applied:

\[ a = \frac{v_f^2 - v_0^2}{2S} \]. Newton’s second law is: \( F = ma \). Substituting for \( a \):

\[ F = \frac{m(v_f^2 - v_o^2)}{2S} \]. Since \( v_0 \) is zero (the ball starts from rest), \( F = \frac{mv_f^2}{2S} \).

Substituting numerical values from the problem,

\[ F = \frac{(0.058 \text{ kg})(45 \text{ m/s})^2}{2(0.44 \text{ m})} = 133 \text{ N} \] (or \( 1.3 \times 10^2 \) N, to two significant figures).

There is an excellent picture in your textbook that goes with this problem, on p. 132. We will use their notation also, to make it easier for you to follow along. Our approach is simple: since the situation has the two forces \( F_{SM} \) and \( F_{EM} \) perpendicular to each other, we can calculate their magnitudes and add them just as we would orthogonal component forces – after all, that’s what they are, really.

\[ F_{SM} = G \frac{mSm_M}{r_{SM}^2} = (6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2) \left(\frac{1.99 \times 10^{30} \text{ kg} \times 7.35 \times 10^{22} \text{ kg}}{1.50 \times 10^{11} \text{ m}^2}\right) = 4.34 \times 10^{-20} \text{ N} \]

\[ F_{EM} = G \frac{mEm_M}{r_{EM}^2} = (6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2) \left(\frac{5.98 \times 10^{24} \text{ kg} \times 7.35 \times 10^{22} \text{ kg}}{3.85 \times 10^8 \text{ m}^2}\right) = 1.98 \times 10^{-20} \text{ N} \]

We can get the total magnitude from Pythagoras’s Theorem:

\[ F = \sqrt{F_{SM}^2 + F_{EM}^2} = \sqrt{(4.34 \times 10^{-20} \text{ N})^2 + (1.98 \times 10^{-20} \text{ N})^2} = 4.77 \times 10^{-20} \text{ N} \]
The two pictures above are free-body diagrams of the climber suspended from the rope. In the left-hand diagram, the three forces acting on her are shown; on the right-hand side, the two tension forces, $T_L$ and $T_R$, have been resolved into x-y components. The climber is at rest, so her velocity is constant (zero, in this case), and Newton’s first law can be applied in each direction.

Based on the angles shown in the drawing, the tension components are:

$$T_{RX} = T_R \sin \theta_R \hspace{1cm} T_{RY} = T_R \cos \theta_R$$

Newton’s first law, in the X direction:

$$\sum F_X = T_R \sin \theta_R - T_L \sin \theta_L = 0 \quad (1)$$

and in the Y direction:

$$\sum F_Y = T_L \cos \theta_L + T_R \cos \theta_R - W = 0 \quad (2)$$

This gives us two equations in two unknowns. Solve equation (1) for $T_R$:

$$T_R = \frac{T_L \sin \theta_L}{\sin \theta_R} \quad (3)$$

Now, substitute this result into equation (2) and solve for $T_L$:

$$T_L \cos \theta_L + \left( \frac{T_L \sin \theta_L}{\sin \theta_R} \right) \cos \theta_R - W = 0$$

$$T_L = \frac{W}{\cos \theta_L + \frac{\sin \theta_L \cos \theta_R}{\sin \theta_R}} = \frac{535 \text{ N}}{\cos 65.0^\circ + \frac{\sin 65.0^\circ \cdot \cos 80.0^\circ}{\sin 80.0^\circ}} = 918.57 \text{ N} = 919 \text{ N}$$

Notice that since we calculated an intermediate numerical result for $T_L$, we elected to “carry” a few extra significant figures into the next calculation, in which we substitute our result for $T_L$ back into equation (3):
This problem is not as difficult as it looks; we just need to bear in mind that, while the two cords may have (and do have) different tensions, all the elements of the system – all three blocks and both cords – share a common acceleration. We draw three free-body diagrams: one for each block.

In order to make progress, we have to say in which direction the whole system will accelerate, so that we know how to orient the kinetic frictional force in the center free-body diagram. Since \( m_3 \) is larger than \( m_1 \), it seems reasonable to say that its larger weight force will cause \( m_2 \) to accelerate from left to right. If we guess wrong, however, at some point the algebra will correct us with an unexpected minus sign. So, we can press on with confidence.

In the left-hand diagram, we apply Newton’s second law: 
\[
\sum F_y = T_1 - m_1 g = m_1 a, \text{ or } T_1 = m_1 (a + g) \tag{1}
\]

In the center diagram, we note that the block will not accelerate in the vertical direction. So, we can apply Newton’s first law:

\[
\sum F_y = N - m_2 g = 0. \text{ That means that } N = m_2 g, \text{ and allows us to write an expression for } F_K:
F_K = \mu_k N = \mu_k m_2 g.
\]

Now, we can apply Newton’s second law in the horizontal direction:

\[
\sum F_x = T_2 - T_1 - F_K = m_2 a. \text{ Substituting for } F_K,
T_2 - T_1 - \mu_k m_2 g = m_2 a \tag{2}
\]
Newton’s second law, applied to the right-hand diagram, says:

\[ \sum F_y = T_2 - m_3g = m_3(-a) \]

Note that we applied a minus sign to \( a \) in the preceding equation. We did so because the acceleration is downward (negative), but we have been using \( a \) to denote the magnitude of an acceleration which has been considered positive up to now – and all parts of the system will have the same acceleration. Solving this equation for \( T_2 \):

\[ T_2 = m_3(g - a) \]  \( (3) \)

Now, have patience – we’re almost finished! The physics is done; what we have left is simply a math problem. Equations (1), (2), and (3) are three equations in three unknowns: \( T_1 \), \( T_2 \), and \( a \). We substitute the right-hand sides of equations (1) and (3) for \( T_1 \) and \( T_2 \) in equation (3), and then solve that equation for \( a \):

\[ T_2 - T_1 - \mu_k m_2g = m_2a \]  \( (2) \)

\[ m_3(g - a) - m_1(a + g) - \mu_k m_2g - m_2a = 0 \]

\[ m_3g - m_3a - m_1a - m_1g - \mu_k m_2g - m_2a = 0 \]

\[ (-m_3 - m_2 - m_1)a = (m_1 + \mu_k m_2 - m_3)g \]

\[ a = \frac{m_1 - m_1 - \mu_k m_2}{m_1 + m_2 + m_3} g = \frac{25.0 \text{ kg} - 10.0 \text{ kg} - 0.100 \cdot 80.0 \text{ kg}}{10.0 \text{ kg} + 80.0 \text{ kg} + 25.0 \text{ kg}} \cdot 9.80 \text{ m/s}^2 = 0.597 \text{ m/s}^2 \]

or 0.60 m/s\(^2\), to two significant figures. Now, we substitute the calculated value of \( a \) back into equations (1) and (3) to obtain \( T_1 \) and \( T_2 \):

\[ T_1 = m_1(a + g) = (10.0 \text{ kg})(0.597 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 104 \text{ N} \]

\[ T_2 = m_3(g - a) = (25.0 \text{ kg})(9.80 \text{ m/s}^2 - 0.597 \text{ m/s}^2) = 230 \text{ N} \]