be of length \( l \). So the concentration formula (17), the substance remains in the
functions for this term-

\[ 1 = \frac{4}{\pi} \left( \sin \frac{\pi x}{l} + \frac{1}{3} \sin \frac{3\pi x}{l} + \frac{1}{5} \sin \frac{5\pi x}{l} + \cdots \right) \]

A quantum-mechanical particle on the line with an infinite potential outside the interval \((0, l)\) ("particle in a box") is given by Schrödinger’s equation \( u_t = iu_{xx} \) on \((0, l)\) with Dirichlet conditions at the ends. Separate the variables and use (8) to find its representation as a series.

Consider waves in a resistant medium that satisfy the problem

\[ u_{tt} = c^2 u_{xx} - ru_t \quad \text{for } 0 < x < l \]
\[ u = 0 \quad \text{at both ends} \]
\[ u(x, 0) = \phi(x) \quad u_t(x, 0) = \psi(x), \]

where \( r \) is a constant, \( 0 < r < 2\pi c/l \). Write down the series expansion of the solution.

Do the same for \( 2\pi c/l < r < 4\pi c/l \). Write down the series expansion of the solution.

Separate the variables for the equation \( iu_t = u_{xx} + 2u \) with the boundary conditions \( u(0, t) = u(\pi, t) = 0 \). Show that there are an infinite number of solutions that satisfy the initial condition \( u(x, 0) = 0 \). So uniqueness is false for this equation!

**4.2 THE NEUMANN CONDITION**

The same method works for both the Neumann and Robin boundary conditions (BCs). In the former case, (4.1.2) is replaced by \( u_x(0, t) = u_x(l, t) = 0 \). Then the eigenfunctions are the solutions \( X(x) \) of

\[ -X'' = \lambda X, \quad X'(0) = X'(l) = 0, \]

other than the trivial solution \( X(x) \equiv 0 \).

As before, let’s first search for the positive eigenvalues \( \lambda = \beta^2 > 0 \). As in (4.1.6), \( X(x) = C \cos \beta x + D \sin \beta x \), so that

\[ X'(x) = -C \beta \sin \beta x + D \beta \cos \beta x. \]